RECEIVERS FOR PULSED RADAR WITH RANDOM DIPOLE MODELLED CLUTTER

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CERTIFICATE

This is to certify that the thesis entitled 'RECEIVERS FOR A PULSED RADAR WITH RANDOM DIPOLE MODELLED CLUTTER' by B. Venkataramani has been carried out under my supervision and that it has not been submitted elsewhere for a degree.

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TABLE OF CONTENTS

ABSTRACT

In this thesis, two structured receivers for a pulsed radar for detecting a Swerling 1 target in the presence of a rotating random dipole modelled clutter are considered. The clutter model incorporated is representative of chaff and vegetation clutter characterised by spectrum with a slow secular component. The structured receivers considered are the conventional receiver and the mismatched receiver. The performances of these receivers are evaluated for 4 and 16 hits/scan and they are compared with the performance obtainable with a white-noise optimum receiver (in the absence of clutter). Their performances, for the case of 4 hits/scan are also compared with that of the optimum receiver which is difficult to implement. The study confirms that for targets with large doppler, the degradation in performance of the sub-optimum receiver is small. For a slowly changing clutter environment, the requirements of an adaptive version of the mismatched receiver are worked out.

CHAPTER-1

INTRODUCTION :

The signal received by a Radar due to its own transmission may be considered to consist of two components one arising from the scattering of electromagnetic waves from the desired target when present and the other due to returns from all other objects illuminated by the radar beam. The latter component is referred to as clutter and can be further classified as ground clutter, sea clutter, second time around clutter, precipitation clutter, angel clutter and so on. The characteristics of clutter depend not only on the relevant properties of the objects under consideration but also on the Radar system parameters such as frequency, polarisation and bandwidth etc.

It is well known that the optimum receiver for the detection of a target in the presence of thermal noise alone, consists of a filter matched to the signal return from the target [1]. In the presence of clutter, the performance of the above white-noise optimum receiver-usually called the conventional receiver can be considerably degraded. But the design and implementation of receivers that are optimum for the detection of targets in the presence of clutter are

rendered difficult by two features that are typical of many engineering situations. The first of them concerns the absence of reliable statistical models for the clutter. The second relates to the practical difficulties that are encountered in finding the solution to the associated integral equation even when the statistical characteristics of the target and clutter are available. Therefore in practice efforts are directed to the design of structured sub-optimum receivers with differing degrees of adaptation to and assumptions about clutter characteristics.

For the design of both optimum and suboptimum receivers when the characteristics of the clutter spectrum are not known a priori, they are quite often estimated on line. For estimating the clutter spectrum some appropriate parametric model is assumed and the model parameters are estimated on line.

The clutter samples are modelled as an M parameter Autoregressive (AR) process and these M parameters are estimated by applying a discrete Kalman filtering algorithm to the message and observation model in Bowyer [26]. The maximum entropy methods are used to estimate the clutter spectrum in Viswesvariah [14] . A detailed survey and list of references of this method are contained in [14]. Wong, Reed et. al. [23] developed a clutter model assuming the clutter

to be due to a collection of rotating random dipoles and this model was further studied by Haykins [17] and the details of a practical receiver based on this clutter model is reported in [19]. However the ROC is not obtained for this receiver.

In this thesis, we assume Wong's model for clutter and consider some receiver schemes for rejection of such clutter and compute and compare their performances in the Chapters to follow.

ORGANIZATION OF THE THESIS:

After a brief review of the previous related work in Chapter 1, Chapter 2 discusses the clutter, target models used in this thesis. The optimum receiver problem is formulated and the various integral equation methods are briefly reviewed. In Chapter 3 the optimum receiver problem is considered assuming the transmitted signal to be rectangular pulse train. An expression for the performance index of the optimum receiver is obtained. In Chapter 4 some relatively easily implementable receivers are considered. The receivers considered are the conventional receiver, the mismatched receiver and the discrete resolution receiver. In each case, expressions for the performance index are derived. Further,

for the mismatched receiver an adaptive scheme is studied in some detail. A quantization scheme along with a procedure for the clutter model parameters is considered. In Chapter 5 the performances of the various receivers proposed are compared. The appropriate assumptions involved in the numerical computations are discussed. Chapter 6 provides a summary of the studies carried out and some suggestions for future work. A list of programmes used is given in the Appendix A.

CHAPTER-2

TARGET, CLUTTER MODEL AND PROBLEM FORMULATION

Target detection performance of a radar receiver evidently depends on the electromagnetic scattering properties of the target and its environment. But because of the complexity and uncertainties involved in obtaining such information, radar receivers are often designed to maximize the probability of target detection in the presence of clutter and thermal noise for a given probability of false alarm that can be tolerated. In such an approach a statistical description of the radar echoes from the target and its environment is adequate for receiver design [3]. In the following, we provide, to serve as a ready reference, a brief summary of the radar target and clutter models which are utilized in the design of receivers proposed in this thesis. Following this, we give a preliminary account of the target detection problem and indicate the nature of the dependence of its solution on the statistical characteristics of the target and clutter.

2.1 TARGET MODEL.

A convenient and often used classification of radar echoes or equivalently of radar targets is provided by their fading and time (delay) and /or frequency (doppler) spread characteristics relative to the time duration and bandwidth

of the transmitted waveform. When the width of the received pulse returned by the target is approximately same as the transmitted pulse i.e. when there is no spreading of the pulse due to the target, the target can be assumed to be a point target. Further if the fluctuation rate of the target is sufficiently small so that over a period of time equal to the pulsewidth of the transmitted signal, the target can be assumed to be stationary then the target can be modelled as a slowly fluctuating point target. The above conditions underwhich a target can be modelled as a slowly fluctuating point target can also be expressed in terms of the relative magnitude of the time duration, bandwidth of the transmitted pulse compared to the target characteristics [4].

For the transmitted signal normally used in a pulsed radar the aircraft can be modelled as a slowly fluctuating point target [6]. So we shall consider some statistical models for the slowly fluctuating point target [7], [24].

Since the radar cross section (RCS) of a target is a measure of the proportion of the incident energy transmitted back to the radar, the targets can be modelled in terms of their RCS. The statistical models for the radar cross section for the slowly fluctuating point target [ ] is given in table 2.1.

Table 2.1

Statistical models for the RCS of slowly fluctuating point targets.

Target Model	Probability density of RCS	Fluctuation Rate
Swerling 1	$\frac{1}{\bar{x}} \exp\left(-\frac{x}{\bar{x}}\right)$ for $x \geq 0$	Scan-Scan
Swerling 2	"	Pulse-Pulse
Swerling 3	$\frac{4x}{\bar{x}^2} \exp\left(\frac{-2x}{\bar{x}}\right)$ for $x \geq 0$	Scan-Scan
Swerling 4	"	Pulse-Pulse

Swerling 1 and 2 target corresponds to the case when the target consists of a large number of scatterers of equal echoing area.

Swerling 3,4 corresponds the case when there is a large reflector in addition to a number of small reflectors.

For Swerling 1,3 targets the RCS of the target can be assumed to be constant over a period of one scan but it varies from one scan to another. In Swerling 2,4 targets the RCS for successive pulses are independent.

We shall assume a Swerling 1 target in this thesis. Hence the RCS of the target remains constant over one scan period. If we denote the complex random multiplier which relates the transmitted signal to the received signal component due to the target as $\tilde{b}$ then the envelope $/\tilde{b}/$ is a Rayleigh random variable whose moments are

$$E[\,|\tilde{b}|\,] = \sqrt{\frac{\pi}{2}}\,\sigma_b \qquad (2.1)$$

$$E[\,|\tilde{b}|^2] = 2\sigma_b^2 \qquad (2.2)$$

The value of σ_b^2 usually includes the antenna gains and path losses in addition to the RCS of the target [3].

2.3 CLUTTER MODEL.

The clutter can be modelled as a doubly dispersive target for the transmitted signal whose time duration is usually small compared to the reciprocal of the bandwidth of the scattering function of the random scatterers and the bandwidth is large compared to the reciprocal of the length of the scattering area. Unlike the target return the interference or the clutter return is due to objects located at different ranges.

If we transmit a signal whose complex envelope is $\tilde{f}(t)$ the reflected signal from the range interval $(\mu, \mu+d\mu)$ is given by

$$\tilde{n}_c(t, \mu) = \sqrt{Et}\ \tilde{f}(t-\mu)\,\tilde{b}(t-\frac{\mu}{2}, \mu)\,d\mu \qquad (2.3)$$

where $\tilde{b}(t, \mu)$ is the reflection process which is normally considered to be complex gaussian process. The return from the entire range interval is

$$\tilde{n}_c(t) = \int_{\alpha}^{\alpha} \sqrt{E_t}\,\tilde{f}(t-\mu)\tilde{b}(t-\tfrac{\mu}{2},\mu)d\mu \tag{2.4}$$

$\tilde{n}_c(t)$ is considered normally to be a sample function from a zero mean complex gaussian process. It can be characterized by the covariance function

$$\tilde{K}_{n_c}(t,u)=E\left[\iint E_t\,\tilde{f}(t-\mu)\tilde{f}^*(u-\mu_1)\tilde{b}(t-\tfrac{\mu}{2},\mu)\,\tilde{b}^*(u-\tfrac{\mu_1}{2},\mu_1)d\mu d\mu_1\right] \tag{2.5}$$

$$= E_t\iint \tilde{f}(t-\mu)\left\{E\left[\tilde{b}(t-\tfrac{\mu}{2},\mu)\tilde{b}^*(u-\tfrac{\mu_1}{2},\mu_1)\right]\right\}\tilde{f}^*(u-\mu_1)d\mu d\mu_1 \tag{2.6}$$

If we make two assumptions :

1) The returns from different intervals are statistically independent.

2) The return from each interval is a sample function of a stationary, zero mean, complex gaussian random process. Then we can write.

$$E[\tilde{b}(t,\mu)\tilde{b}^*(u,\mu_1)]=\tilde{K}_{DR}(t-u,\mu)\delta(\mu-\mu_1) \tag{2.7}$$

The function $\tilde{K}_{DR}(\tau,\mu)$ is a two variable function that depends on the reflective properties of the clutter. The Fourier transform of $\tilde{K}_{DR}(\tau,\mu)$ is defined as the scattering function $\tilde{S}_{DR}(f,\mu)$

(i.e.) $$\tilde{S}_{DR}(f,\mu)=\int \tilde{K}_{DR}(\tau,\mu)\,e^{-j2\pi f\tau}d\tau \tag{2.8}$$

Physically $\tilde{S}_{DR}(f, \mu)$ represents the spectrum of the process $\tilde{b}(t, \mu)$. Substituting (2.7) in (2.6)

$$\tilde{K}_{n_c}(t, u) = E_t \int_{-\infty}^{\infty} \tilde{f}(t-\mu)\, \tilde{K}_{DR}(t-u, \mu)\, \tilde{f}^*(u-\mu)\, d\mu \qquad \ldots . \quad (2.9)$$

For different clutter $\tilde{K}_{DR}(\tau, \mu)$ will be different.

RANGE INVARIANT SCATTERING FUNCTION

We want to consider the specific clutter for which the value of the scattering function given by (2.8) at any frequency shift f is the same for all ranges.

$$\text{(i.e.)} \quad \tilde{S}_{DR}(f, \mu) = \tilde{S}_{DR}(f) \quad -\infty < \mu < \infty \qquad \ldots . \quad (2.10)$$

Hence the scattering function has uniform doppler profile. We can treat the scattering function to be infinite in length if the uniform doppler profile extends beyond the range of a possible target by L where L is the length of the target [5].

When the clutter is infinite in extent, the clutter return is a sample function of a stationary process. Using (2.10) in (2.9)

$$\tilde{K}_{n_c}(t, \mu) = E_t \iint_{-\infty}^{\infty} \tilde{f}(t-\mu)\tilde{f}^*(u-\mu) \times \tilde{S}_{DR}(f)\, e^{j2\pi f(t-u)}\, df\, d\mu$$

$$= E_t \int_{-\infty}^{\infty} \tilde{f}(t-\mu)\, \tilde{f}^*(u-\mu)\, \tilde{K}_{DR}(t-u)\, d\mu$$

$$= E_t\, \tilde{K}_{DR}(t-u) \int_{-\infty}^{\infty} \tilde{f}(t-\mu)\, \tilde{f}^*(u-\mu)\, d\mu \qquad \ldots . (2.11)$$

$$= E_t \tilde{K}_{DR} (t-u) \int_{-\infty}^{\infty} \tilde{f}(t+\tau) * \tilde{f}^*(t')\, dt' \qquad \text{.... (2.12)}$$

where $t-\mu = t'+\tau$ .....(2.13)

and $u-\mu = t'$ ..... (2.14)

From (2.13) and (2.14) $\tau = t-u \quad dt' = d\mu$

But since the autocorrelation function of the transmitted signal $\tilde{R}(\tau) = \int_{-\infty}^{\infty} \tilde{f}(t'+\tau) * \tilde{f}^*(t')\, dt'$ ..... (2.15)

(2.12) can be written as

$$\tilde{K}_{n_c}(t,u) = E_t \; \tilde{K}_{DR}(t-u) * R(t-u) \qquad \text{..... (2.16)}$$

We have derived the covariance function corresponding to the clutter with range invariant scattering function in detail since the clutter model that use in this thesis has this characteristics.

2.4 THE RANDOM DIPOLE CLUTTER MODEL.

Wong et al. [25] developed a clutter model assuming the clutter signal to be due to a collection of scatterers considered to be rotating random dipoles. The basis for the above model can be described as follows. When an RF pulse is transmitted from a radar toward a cloud of random scatterers moving about and reflecting energy independently of one another and if in addition the cloud is assumed to have an overall drift velocity, the

echoes returned to the radar will arrive at a rate which depends upon the local density of the cloud. If the effects of multiple scattering and scatterer rotation are neglected the echo signal from a particular scatterer can be regarded as a Doppler shifted replica of the transmitted waveform. However, any change in the orientation of the scatterers can cause variations in the returned echo power and phase. Hence the effect of scatterer rotational motion is included in calculating the echo return.

The time varying correlation function of the echo signal is derived in terms of the characteristics of the transmitted waveform, polarization and the distribution of the scatterers. For the case of linear transmit-linear receive polarization and the transmitted signal consisting of a coherent pulse train, if the echo rate varies only slowly with time and if the observation interval is long compared with signal duration (so that the clutter process becomes stationary) then the normalized autocorrelation function of the echo signal (clutter signal) is given by [22]

$$K_{n_c}(t-u)= \frac{2}{3}\left[1+\tfrac{1}{2} \cos a\tau e^{-b\tau^2}\right] e^{-d\tau^2+jc\tau} R(\tau) \qquad (217)$$

where

$$\begin{aligned} a &= 4\pi V_{or} \\ b &= 8\pi^2 \sigma_r^2 \\ c &= 2\pi V_{od} \\ d &= 2\pi^2 \sigma_d^2 \end{aligned}$$

In this model the scatterers are treated as dipoles instead of points with variable cross sections. The dipole rotation frequency and the doppler shift are assumed to be gaussian distributed. V_{or}, σ_r^2 are the mean, variance of dipole rotation frequency V_{od}, σ_d^2 are the mean, variance of doppler shift respectively.

Though this model is derived assuming a 'chaff cloud' this model is equally valid for returns from branches, leaves, grass etc. having a rotational motion under the effects of wind forces.

Comparing (2.17) with (2.16) this particular clutter has a range invariant scattering function with

$$K_{DR}(\tau) = \frac{2}{3}\left(1+\tfrac{1}{2}\cos a\tau e^{-b\tau^2}\right) e^{jc\tau - d\tau^2} \quad (2.18)$$

we shall be using this model for the clutter in this thesis. The advantage with this model is that it is enough to know just four parameters to know the clutter characteristics.

2.5 TYPICAL VALUES FOR THE CLUTTER MODEL PARAMETERS.

In order to obtain some typical values for the clutter model parameters we have to specify both the interpulse period T_P and the operating frequency f_o. The maximum values for these model parameters are obtained by Haykins et al [18], Barton [25]. When the operating frequency is 1GHz the typical maximum values for the model parameters are :

$$\sigma_{dmax} = \frac{0.044}{T_P} \text{ m/s} \qquad \sigma_{rmax} = \frac{0.033}{T_P} \text{ Hz}$$

$$V_{odmax} = \frac{0.1}{T_P} \text{ m/s} \qquad V_{ormax} = \frac{0.033}{T_P} \text{ Hz}$$

For a PRF of 540 Hz the maximum values for a,b,c,d

$$a = 4\pi V_{or} \cong 216 \quad \text{radians/sec}$$

$$b = 8\pi^2 \sigma_r^2 \cong 25720 \quad ''$$

$$c = 2\pi V_{od} \cong 324 \quad ''$$

$$d = 11130 : (2\pi^2 \sigma_d 2) \quad ''$$

2.6 PROBLEM FORMULATION.

In the hypothesis - testing problem of detecting a radar target the hypotheses are :

$$\tilde{r}(t) = \tilde{B} \; \tilde{f}_s(t) + \tilde{n}_c(t) + \tilde{w}(t) \qquad : H_1$$

$$= \tilde{n}_c(t) + \tilde{w}(t) \qquad : H_0$$

where $\tilde{r}(t)$ is the complex envelope of the received waveform, $\tilde{f}_s(t)$ is the delayed, doppler shifted replica of the transmitted signal, $\tilde{B}$ is the complex gaussian random variable multiplier due to the target, $\tilde{n}_c(t)$ is the return due to the clutter and $\tilde{w}(t)$ is the thermal noise assumed to be white and which includes the receiver noise.

The optimum receiver correlation waveform $\tilde{g}(t)$ which maximizes the probability of target detection in the presence of clutter and thermal noise satisfies the integral equation [5].

$$\tilde{f}_s(t) = \int_{Ti}^{T_f} K_{\tilde{n}}(t,u)\, \tilde{g}(u)\, du \qquad T_i \leq t \leq T_f$$

where $\tilde{n}(t) = \tilde{n}_c(t) + \tilde{w}(t)$

and $K_{\tilde{n}}(t,u) = E[\, \tilde{n}(t)\, \tilde{n}^*(u)\,]$

$$= \tilde{K}_{n_c}(t,u) + No\,\delta(t-u)$$

where $\tilde{K}_{n_c}(t,u) = E[\, \tilde{n}_c(t)\, \tilde{n}_c^*(u)\,]$

and $E[\, \tilde{w}(t)\, \tilde{w}^*(u)\,] = No\,\delta(t-u)$

where No is the power spectral density of white noise.

The decision is taken by computing the sufficient statistic $\tilde{l} \triangleq \int_{T_i}^{T_f} \tilde{r}(t)\, \tilde{g}^*(t)\, dt$ and then comparing $|\tilde{l}|^2$ with a threshold. We want to consider the above hypotheses testing problem when the clutter covariance kernel is given by (2.17). This covariance Kernel is derived assuming the transmitted signal to be a pulse train. We shall consider the optimum receiver in chapter 3 and sub optimum receivers in chapter 4. Since we have to solve the integral equation in

eqn.(2.22) for arriving at the optimum receiver we shall briefly review the solution techniques for the integral equations.

Substituting (2.25) in (2.22) we get

$$\tilde{f}_s(t) = \int_{T_i}^{T_f} \tilde{K}_{n_c}(t,u)\,\tilde{g}(u)\,du + No\,\tilde{g}(t) \quad \ldots \quad (2.26)$$

This is a Fredholm equation of the second kind.

2.7 REVIEW OF METHODS FOR SOLUTION OF FREDHOLM INTEGRAL EQUATIONS.

DEGENERATE KERNEL METHOD.

When the Kernel of the integral equation (2.22) is seperable into functions of t alone by u alone then the integral equation can be reduced to an algebraic equation [15]. Solving the algebraic equation, The solution for the integral equation can be obtained.

STATE VARIABLE APPROACH.

Some integral equations can be solved by deriving a differential equation. Since the differential equations are easy to solve the solution for the integral equation can be obtained by proper substitution [2]. For this to be possible the random process characterized by the Kernel should permit state space representation. For the state space

representation to be possible the Kernel has to be seperable in the variables t and u [1].

ORTHOGONAL SERIES EXPANSION METHOD.

If the eigen values and eigen functions of the Kernel of the integral equation can be found then the solution to the integral equation can be expressed in terms of these eigen values and eigen functions [1].

METHOD OF RESOLVENT KERNELS.

The Resolvent Kernel of a Kernel is given by

$R_K(t,\tau) = \sum_{n=1}^{\infty} K_{in}(t,\tau)\,(-1)^{n-1}$ where $K_{in}(t,\tau)$ is the iterated Kernel given by $K_{in}(t,\tau) = \int_{T_i}^{T_f} K_m(t,s)\, K_{n-m}(s,\tau)ds$ and $K_o(t,\tau) = K_{n_c}(t-\tau)$. In terms of the resolvent Kernel the solution becomes [19]

$$g(t) = f_s(t) + \int_{T_i}^{T_f} R_K(t,\tau)\, f_s(\tau)\, d\tau$$

FOURIER TRANSFORM METHOD.

In the Integral equation if we assume that the integration limits can be extended to infinity the Fourier Transform of both sides of equation (2.5) can be taken. If we define

$$F_s(f) \triangleq \int_{-\infty}^{\infty} f_s(t)\, e^{-jwt} dt$$

$$S_{n_c}(t) \triangleq \int_{-\infty}^{\infty} K_{n_c}(t)\, e^{-jwt}\, dt$$

$$G(f) \triangleq \int_{-\infty}^{\infty} g(t)\, e^{-jwt}\, dt$$

Then $$G(f) = \frac{F_s(f)}{S_{n_c}(f)+No}$$

If the inverse Fourier transform of $G(f)$ can be found then $g(t)$ is known.

CHAPTER-3

THE OPTIMUM RECEIVER

In the previous chapter we had indicated that an integral equation has to be solved to obtain the optimum receiver correlation waveform. We reviewed various methods for solving the integral equation. Further we also described the target and clutter models that are of interest to this thesis. To solve the integral equation, we need to specify the transmitted waveform. In this study we assume that the transmitted waveform shown in Fig.3.1(a) is a train of M number of equispaced rectangular pulse with pulsewidth T and interpulse period T_p.

3.1 AUTOCORRELATION OF THE TRANSMITTED WAVEFORM.

The transmitted signal $f_s(t)$ and the autocorrelation function $R(\tau)$ can be written as

$$f_s(t)= \sum_{m=0}^{M-1} f(t-mT_p) \qquad \text{.... (3.1)}$$

$$\text{where} \quad f(t)= \frac{1}{\sqrt{T}} \quad \text{for} \quad 0\leq t\leq T \qquad \text{.... (3.2)}$$

$$= 0 \quad \text{otherwise}$$

$$R(\tau)= \sum_{k=0}^{M-1} \frac{M-k}{M} \left(1- \frac{\beta(\tau_k)}{T} \right) \qquad \text{.... (3.3)}$$

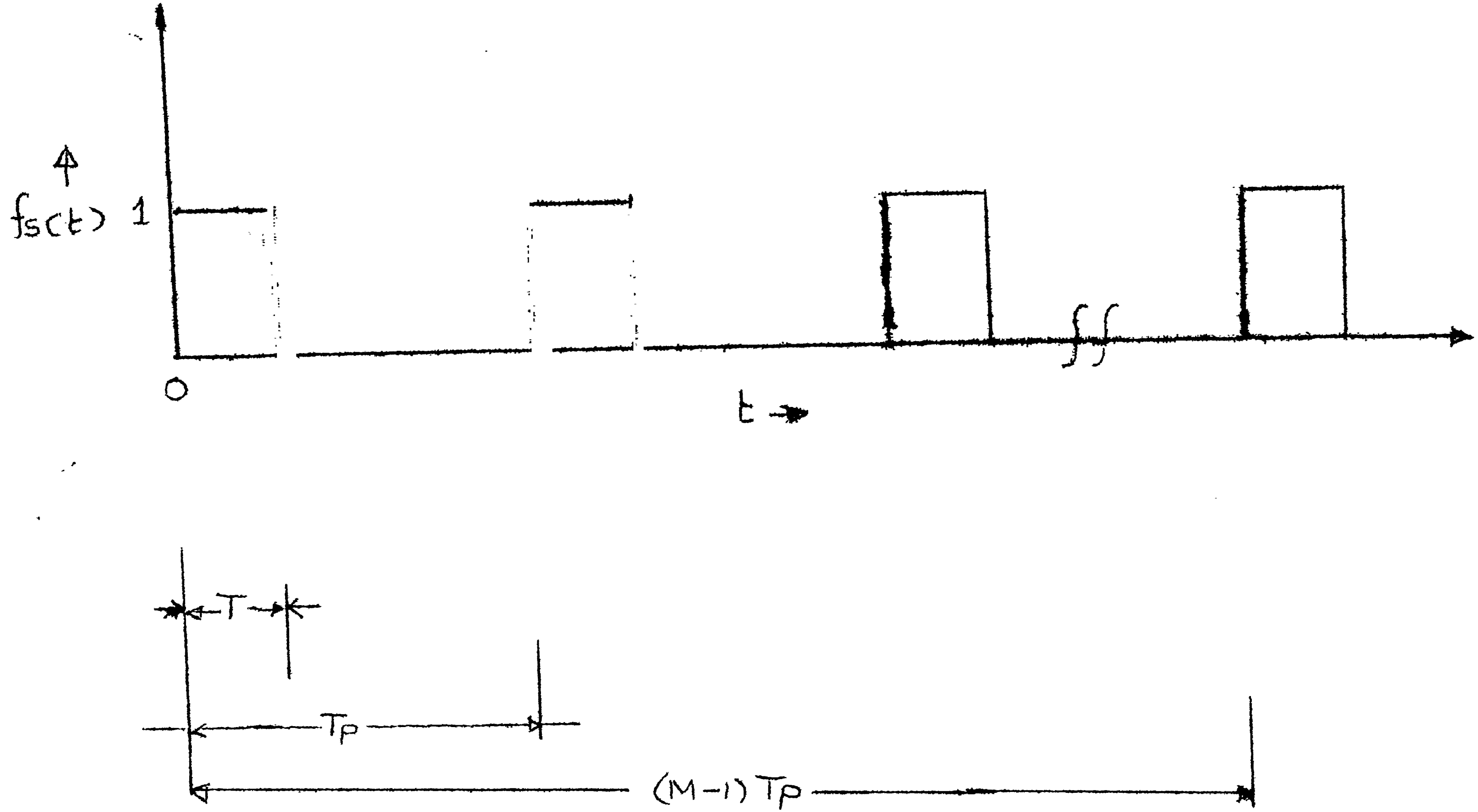


FIG 3.1 (a) TRANSMITTED PULSE TRAIN

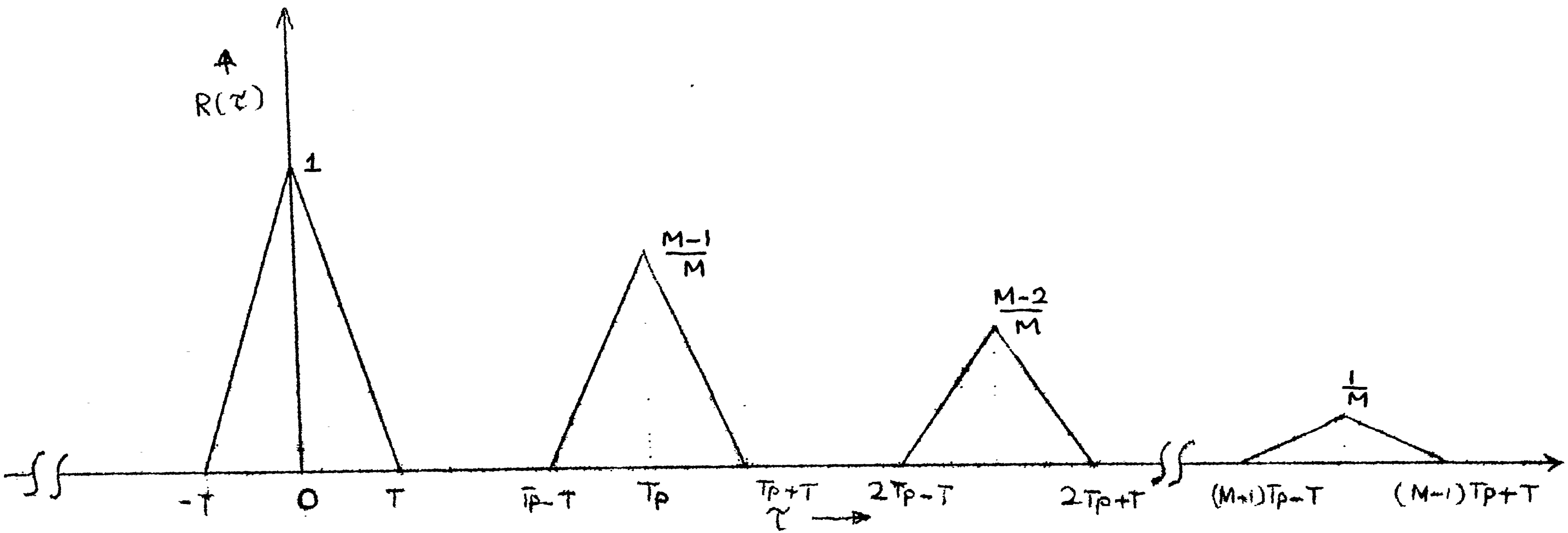


FIG 3.1 (b) AUTOCORRELATION FUNCTION OF THE TRANSMITTED PULSE TRAIN

where $\beta(\tau_k) = \beta(\tau - kT_p)$

$$= 0 \quad \text{for } |\tau - kT_p| < T \qquad \ldots. (3.4)$$

$$= |\tau - kT_p| \quad \text{otherwise}$$

For $M=2$

$$R(\tau) = \sum_{k=0}^{1} \frac{2-k}{2}\left(1 - \frac{\beta(\tau_k)}{T}\right)$$

$$= 1 - \frac{\beta(\tau)}{T} + \tfrac{1}{2}\left(1 - \frac{\beta(\tau - T_p)}{T}\right)$$

$$= 1 - \frac{|\tau|}{T} \quad \text{when } |\tau| \leq T \qquad \ldots. (3.5)$$

$$= \tfrac{1}{2}\left(1 - \frac{|\tau - T_p|}{T}\right) \text{ when } |\tau - T_p| \leq T$$

3.2 THE KERNEL OF THE INTEGRAL EQUATION.

The covariance function of the clutter signal can be obtained by substituting the value of $R(\tau)$ from (3.3) and the value of $K_{DR}(\tau)$ from (2.18) into (2.16). We note that the covariance function of the clutter return is also the Kernel of the integral equation in 2.25. If f_D is the doppler shift due to the target then (2.25) becomes

$$\sum_{m=0}^{M-1} f(t - mT_p) e^{j w_D t} = \int_{T_i}^{T_f} \tilde{K}_{n_c}(t-u) \tilde{g}(u) du + N_o \tilde{g}(t) \qquad \ldots.(3.6)$$

For simplicity we shall consider the single pulse case ($M=1$) first. Then the Kernel of the integral equation (3.6) becomes

$$\tilde{K}_{n_c}(t,u)= \frac{2}{3} (1+\tfrac{1}{2}\cos a(t-u)e^{-b(t-u)^2})\exp(jc(t-u) -d(t-u)^2)\times$$

$$(1- \frac{|t-u|}{T})\beta(t-u) \qquad \ldots. \quad (3.6(a))$$

where $\beta(t-u)=1$ if $|t-u|\leq T$

$$= 0 \text{ otherwise} \qquad \ldots. \quad (3.6(b))$$

Since $K_{n_c}(t-u)$ has quadratic terms in the exponent ($(t-u)^2$term) and has the factor $(1- \frac{|t-u|}{T})$ it is not seperable in the variables t and u. Hence this is not a degenerate kernel. Since the Kernel is not seperable the state variable approach, and the orthogonal series expansion methods also fail. For the Kernel given by (3.6a) the iterated Kernels are difficult to find and hence the resolvent Kernel method cannot be obtained. We must find the Fourier transform of the Kernel in order to apply the Fourier transform method to obtain the solution of the integral equation.

3.3 CLUTTER POWER SPECTRAL DENSITY.

To simplify the notation we shall drop the tilde sign for indicating complex waveform. The clutter power spectral density $S_{n_c}(f)$ is given by

$$S_{n_c}(f) = \int_{-\infty}^{+\infty} K_{n_c}(\tau)e^{-jw\tau}d\tau \qquad \ldots. \quad (3.7)$$

Putting $t-u=\tau$ in (3.6a) and substituting in (3.7)

we get

$$S_{n_c}(f)=\int_{-\infty}^{\infty}\frac{2}{3}(1+\tfrac{1}{2}\cos a\tau e^{-b\tau^2})e^{-d\tau^2+jc\tau}\beta(\tau)(1-\frac{|\tau|}{T})e^{-jw\tau}d\tau \quad \ldots. (3.8)$$

$$=\int_{-\infty}^{\infty}[e^{-d\tau^2+jc\tau}+\frac{1}{4}e^{-(b+d)\tau^2+j(a+c)\tau}+\frac{1}{4}\exp(-(b+d)\tau^2+j(c-a)\tau)](1-\frac{|\tau|}{T})e^{-j\omega\tau}\beta(\tau)d\tau \quad \ldots. (3.9)$$

neglecting the normalization constant $\frac{2}{3}$ and expanding.

Since the term inside the square bracket of (3.9) are similar excepting for the multiplying constants in the exponents let us concentrate on one term. Once we obtain the integral for the first term the other integrals can be obtained by simple substitution.

$$\text{Let } S'_{n_c}(f)=\int_{-\infty}^{\infty}e^{-d\tau^2+jc\tau-jw\tau}(1-\frac{|\tau|}{T})\beta(\tau)d\tau \quad \ldots. (3.10)$$

For evaluating $S'_{n_c}(f)$ we shall consider two methods.

3.4 FREQUENCY DOMAIN APPROACH TO EVALUATE THE CLUTTER SPECTRUM.

We can show that

$$\int_{-\infty}^{\infty}(1-\frac{|\tau|}{T})\beta(\tau)e^{-jw\tau}d\tau=T(\frac{\sin\pi fT}{\pi fT})^2=S(f) \quad \ldots. (3.11)$$

$$\text{Now let } S'_{DR}(f)=\int_{-\infty}^{\infty}e^{-d\tau^2+jc\tau}e^{-jw\tau}dt \quad \ldots. (3.12)$$

Then $S'_{DR}(f) = \int_{-\infty}^{\infty} e^{-d\tau^2 - j(w-c)\tau} d\tau$

Completing the square in the exponent we get

$$S'_{DR}(f) = \exp\left[-\frac{(w-c)^2}{4d}\right] \int_{-\infty}^{\infty} \exp\left[-d\left(\tau + j\frac{w-c}{2d}\right)^2\right] d\tau \quad \ldots. (3.13)$$

Since $\int_{-\infty}^{\infty} e^{-\alpha x^2} dx = \sqrt{\frac{\pi}{\alpha}}$

(3.13) becomes

$$S'_{DR}(f) = \sqrt{\frac{\pi}{d}}\, e^{-\frac{(w-c)^2}{4d}} \quad \ldots. (3.14)$$

$$\text{Now } K'_{n_c}(\tau) = \int_{-\infty}^{+\infty} S'_{n_c}(f) e^{jw\tau} df \quad \ldots. (3.15)$$

Comparing (3.7) with (3.8)

$$K'_{n_c}(\tau) = e^{-d\tau^2 + jc\tau}\left(1 - \frac{|\tau|}{T}\right)\beta(\tau) \quad \ldots. (3.16)$$

Since $K'_{n_c}(\tau)$ is the product of 2 time functions $\beta(\tau)\left(1 - \frac{|\tau|}{T}\right)$ and $e^{-d\tau^2+jc\tau}$ $S'_{n_c}(f)$ is the convolution of the Fourier transform of $e^{-d\tau^2+jc\tau}$ with the Fourier transform of $\left(1 - \frac{|\tau|}{T}\right)\beta(\tau)$

From (3.11) and (3.12)

$$S'_{n_c}(f) = \int_{-\infty}^{\infty} S'(x) S'_{DR}(f-x) dx$$

$$= \int_{-\infty}^{\infty} T\left(\frac{\sin \pi x T}{\pi x T}\right)^2 \sqrt{\frac{\pi}{d}}\, e^{-\frac{(w-2\pi x-c)^2}{4d}} dx \quad \ldots. (3.17)$$

Since the integrals $\int \frac{\sin x}{x}\,dx$ and $\int e^{-\alpha x^2}dx$ are evaluated using contour integrals we investigated whether contour integral can be used to evaluate (3.17). Eqn.(3.17) can be rewritten as

$$S'_{n_c}(f) = \int_{-\infty}^{+\infty} T\,\frac{(1-\cos 2\pi xT)}{(\pi xT)^2}\,\sqrt{\frac{\pi}{d}}\;e^{-(f-x+fo)^2 k}dx \qquad \ldots.(3.18)$$

To evaluate (3.18) using contour integration we considered the complex function

$$\phi(z) = e^{-K(z-f+fo)^2}\,\frac{(1-e^{i2\pi zT})}{\pi^2 z^2 T} \qquad \ldots.\ (3.19)$$

where z is a complex variable. $\phi(z)$ has a single pole at z=0 and there is no other pole. We investigated evaluating the integral using circular contour first and then using rectangular contour. In both the cases the integral could not be evaluated since the integral is undefined over some segment of the contour [9], [11].

3.5 INTEGRATION AND DIFFERENTIATION OF ERROR FUNCTION.

By integrating and differentiating the error function

$$\operatorname{erf}(t) = \frac{2}{\sqrt{\pi}} \int_0^t e^{-x^2}dx \qquad \ldots.\ (3.20)$$

and $\int_{0}^{\infty} \cos bx e^{-\frac{ax^2}{2}} dx = \sqrt{\frac{\pi}{2a}}\, e^{-b^2/2a}$ (3.21) certain integrals involving e^{-x^2} can be evaluated [16]. For evaluating (3.18) we should consider the integrals

$$\int \frac{e^{-k(x-f+fo)^2}}{x^2}\,dx \text{ and } \int \frac{e^{-k(x-f+fo)^2}}{x^2} \cos bx \, dx \quad \ldots. \; (3.27)$$

By differentiating and integrating (3.20) and (3.21) we cannot obtain the integrals of the form given in (3.27). Hence this method is not suitable.

3.6 TIME DOMAIN APPROACH.

We shall consider the method of obtaining $S'_{n_c}(f)$ from $K'_{n_c}(\tau)$ directly instead of first finding $S(f)$, $S'_{DR}(f)$ and then convolving them together.

Let us rewrite (3.10) as

$$S'_{n_c}(f) = \int_{-\infty}^{\infty} \left(1 - \frac{\tau}{T}\right) \beta(\tau) e^{-D\tau^2 - jw\tau + jc\tau} d\tau \quad (3.28)$$

to avoid confusion of the differential $d\tau$ with the exponent $d\tau^2$ in (3.10).

Since $\beta(\tau)=0$ for $|\tau| > T$ by (3.6b)

$$S'_{n_c}(f) = \int_{-T}^{0} (1+\tau/T) e^{-D\tau^2 - j(w-c)\tau} d\tau \quad \ldots. \; (3.29)$$

$$+ \int_0^T (1-\tau/T)e^{-D\tau^2-j(w-c)\tau}\,d\tau$$

$$= 2R.P\left[\int_0^T (1-\tau/T)e^{-D\tau^2-jw'\tau}\,d\tau\right] \quad \ldots. (3.30)$$

where $w' = w-c$

Now (3.30) can be rewritten as

$$S'_{n_c}(f)=2R.P\left[\int_0^T \left(1+\frac{jw'}{2DT}\right)e^{-(\tau+jw'/2D)^2 D-\frac{w'^2}{4D}}\,d\tau\right.$$

$$\left.-\frac{1}{T}\int_0^T (\tau+jw/2D)e^{-(\tau+jw'/2D)^2 D-\frac{w'^2}{4D}}\,d\tau\right] \quad \ldots. (3.31)$$

$$\text{Let } I_1 \triangleq \int_0^T (1+jw'/2DT)e^{-(\tau+jw'/2D)^2 D-w'^2/4D}\,d\tau \quad \ldots. (3.32)$$

and $(\tau+jw'/2D)^2 D=z^2$

Then $d\tau=dz/\sqrt{D}$ $\quad \tau=T \Rightarrow z=T\sqrt{D}+jw'/2\sqrt{D}$ $\quad \ldots. (3.33)$

$\tau=0 \Rightarrow z=jw'/2\sqrt{D}$

$$\therefore I_1= \frac{1}{\sqrt{D}}\int_{jw'/2\sqrt{D}}^{T\sqrt{D}+jw'/2\sqrt{D}} e^{-z^2}(1+jw'/2DT)\,dz \quad \ldots. (3.34)$$

$$= \frac{1}{\sqrt{D}}(1+jw'/2DT)\left[\operatorname{erf}(T\sqrt{D}+jw'/2\sqrt{D})-\operatorname{erf}(jw'/2\sqrt{D})\right] \quad \ldots. (3.35)$$

where $\mathrm{erf}(z)=2/\sqrt{\pi}\int_0^z e^{-z^2}dz$ .... (3.36)

is the error function with complex arguement z. The values of erf(z)are tabulated in [13].

Let $I_2=-\frac{1}{T}\int_0^T(\tau+jw'/2D)e^{-(\tau+jw'/2D)^2D-w'^2/4D}\,d\tau$

.... (3.37)

and $z=(\tau+jw'/2D)^2D$

Then $dz=2D(\tau+jw'/2D)d\tau$

when $\tau=0 \quad z=-w'^2/4D$

$\tau=T \quad z=D(T+jw'/2D)^2$ .... (3.38)

$$\therefore I_2=\frac{-e^{-w'^2/4D}}{T}\times\int_{\frac{-w'^2}{4D}}^{D(T+jw'/2D)^2}\frac{1}{2D}e^{-z}dz \quad \text{.... (3.39)}$$

$$=\frac{e^{-\frac{w'^2}{4D}}}{2DT}\left[e^{-z}\right]_{\frac{-w'^2}{4D}}^{D(T+jw'/2D)^2}=\frac{1}{2DT}[e^{-DT^2-jw'T}-1]$$

.... (3.40)

$\therefore S'_{n_c}(f)=2R.P[I_1e^{-w'^2/4D}+I_2]$

$$= \frac{2}{\sqrt{D}} \text{ R.P}[\text{erf}(T\sqrt{D}+jw'/2\sqrt{D})]A - \frac{2}{\sqrt{D}}\left(\frac{w}{2DT}\right)A \; I_m[\text{erf}(T\sqrt{D}+\frac{jw'}{2\sqrt{D}})]$$

$$+ \frac{2}{\sqrt{D}} A\left(\frac{w}{2DT}\right) I_m[\text{erf}(jw'/2\sqrt{D})] - \frac{2A}{\sqrt{D}} \text{ R.P}[\text{erf}(jw/2\sqrt{D})]$$

$$+ \frac{1}{2DT}\left[e^{-DT^2}\cos wT - 1\right] \qquad \text{.... (3.41)}$$

Next let us consider the case when there are two pulses in the transmitted waveform. The autocorrelation function of the transmitted pulse train $R(\tau)$ is given by

$$R(\tau) = 1 - \frac{|\tau|}{T} \quad \text{if } |\tau| < T$$

$$= \frac{1}{2}\left(1 - \frac{|\tau - T_P|}{T}\right) \quad \text{if } |\tau - T_P| < T \qquad \text{.... (3.42)}$$

$$= 0 \quad \text{otherwise}$$

$$\text{Let } S''_{n_c}(f) = \int_{-\infty}^{\infty} R(\tau) e^{jc\tau - D\tau^2 - jw\tau} d\tau$$

$$= \int_{-\infty}^{\infty} R(\tau) e^{-jw'\tau - D\tau^2} d\tau \qquad \text{.... (4.43)}$$

where $R(\tau)$ is given by (3.42).

Substituting (3.42) in (3.43) we get

$$S''_{n_c}(f) = 2\text{R.P}\left[\int_0^T \left(1 - \frac{|\tau|}{T}\right) e^{-jw'\tau - D\tau^2} d\tau + \frac{1}{2}\int_{T_P - T}^{T_P + T} \left(1 - \frac{|T_P - T|}{T}\right)\right.$$

$$e^{-jw'\tau - D\tau^2} d\tau \qquad \text{.... (3.44)}$$

Let $I_3 = \int_{T_P-T}^{T_P} (1-\frac{T_P}{T} + \frac{\tau}{T})e^{-jw'\tau-D\tau^2}\, d\tau$ .... (3.45)

Then $I_3= \int_{T_P-T}^{T_P}(1- \frac{T_P}{T} - \frac{jw'}{2DT})e^{-D(\tau+jw'/2D)^2-w'^2/4D}\, d\tau$

$$+ \int_{T_P-T}^{T_P} (\tau+jw'/2D)e^{-D(\tau+jw'/2D)^2-w'^2/4D}\, d\tau \quad \text{....(3.46)}$$

$$= (1- \frac{T_P}{T} -jw'/2DT)e^{-w'^2/4D}[\mathrm{erf}(T_P\sqrt{D}+jw'/2\sqrt{D})-\mathrm{erf}$$

$$(T_P\sqrt{D}-T\sqrt{D} +jw'/2\sqrt{D})]- \frac{1}{2DT}[e^{-T_P^2D-jw'T_P}-\exp$$

$$[-T_P^2D-T^2D+2TT_PD+jw'T-jw'T_P] \quad \text{.... (3.47)}$$

Since the pulse width $T \ll$ interpulse period T_P

$$I_3=(1- \frac{T_P}{T} \frac{-jw'}{2DT}) [\mathrm{erf}(T_P\sqrt{D}+jw'/2\sqrt{D})-\mathrm{erf}(T_P\sqrt{D}-T\sqrt{D}+jw'/2\sqrt{D})]$$

.... (3.48)

$$- \frac{1}{2DT} e^{-jw'T_P} [1-e^{+jw'T}]$$

Similarly

$$I_4= \int_{T_P}^{T_P+T}(1+ \frac{T_P}{T} - \frac{\tau}{T})e^{-D\tau^2-jw'\tau}\, d\tau$$

$$= \left(1+\frac{T_P}{T}+jw'/2DT\right)\left[\mathrm{erf}(T_P\sqrt{D}+T\sqrt{D}+jw'/2\sqrt{D})-\mathrm{erf}(T_P\sqrt{D}+jw'/2\sqrt{D})\right] \quad \ldots. (3.49)$$

$$+\frac{e^{-jw'T_P}}{2DT}\left[e^{-jw'T}-1\right]$$

$$\therefore S''_{n_c}(f)=2.A.R.P[I_3]+2.A.R.P[I_4]+S'_{n_c}(f)+\frac{1}{2DT}[2\cos wT-2]\cos wT_P \quad \ldots. (3.50)$$

$$= 2A.R.P[\mathrm{erf}(x+jy)]\left(1-\frac{T_P}{T}\right)+\frac{2Ay}{T\sqrt{D}}\,I_m[\mathrm{erf}(x+jy)]$$

$$-A\left(1+\frac{T_P}{T}\right)2.R.P[\mathrm{erf}(x-T\sqrt{D}+jy)]-\frac{2Ay}{T\sqrt{c}}\,I_m[\mathrm{erf}(x+T\sqrt{D}+jy)]$$

$$-A\left(1+\frac{T_P}{T}\right)2.R.P[\mathrm{erf}(x+jy)]+\frac{2Ay}{T\sqrt{D}}\,I_m(\mathrm{erf}(x+jy))-\frac{2A}{2DT}\times$$

$$[\cos w'T_P-\cos w'(T_P-T)]$$

$$+\frac{2A}{2DT}[\cos w'(T_P+T)-\cos w'T_P]+S'_{n_c}(f) \quad \ldots. (3.51)$$

where $x=T_P\sqrt{D}$, $y=w'/2\sqrt{D}$, $A=\emptyset(n)e^{-w'^2/4D}$ and $\emptyset(n)=\frac{M-n}{M}$

For $S''_{n_c}(f)$ $n=1$

$S'^{m}_{n_c}(f)$, the spectrum for any arbitrary m can be found by substituting T_P by mT_P wherever T_P occurs in the expression for $S''_{n_c}(f)$ and replacing $S'_{n_c}(f)$ by $S'^{m-1}_{n_c}(f)$. We observe that

the expressions for $S_{n_c}(f)$ both for a single pulse and multiple pulses are unwieldy. Hence we attempt to obtain an approximate clutter spectrum in the next section.

3.7 APPROXIMATE CLUTTER SPECTRUM.

Approximate expression for $S_{n_c}(f)$ when the transmitted signal consists of a train of equispaced rectangular pulses can be derived as follows : The observation that the clutter is a slowly varying process [23], so that over a period of time T= pulsewidth of the transmitted signal pulses which are of the order of µsec it can be assumed to be constant simplifies the problem.

Let the number of pulses be M. The transmitted signal and the autocorrelation function of it are given in equations (3.1) to (3.5). We observe that $R(\tau)=R(-\tau)$. When the received signal is heterodyned with an oscillator signal of frequency fo+fd where fo,fd are the transmitted frequency, mean doppler frequency of the clutter return respectively

then $$K_{DR}(\tau)=e^{-b\tau^2}+\frac{1}{4}e^{-(b+d)\tau^2-ja\tau}+\frac{1}{4}e^{-(b+d)\tau^2+ja\tau}$$

.... (3.52)

and $K_{DR}(\tau)=K_{DR}(-\tau)$

$$\therefore S_{n_c}(f)= 2.R.P\int_{-\infty}^{\infty} R(\tau)K_{DR}(\tau)e^{-jw\tau}d\tau$$

$$= 2R.P[\int_{0}^{T} K_{DR}(\tau)(1-\frac{\tau}{T})e^{-jw\tau}\,d\tau+\sum_{i=1}^{M-1}[\int_{iT_{P-T}}^{iT_P} K_{DR}(\tau)$$

$$(1+\frac{\tau}{T}-\frac{iT_P}{T})e^{-jw\tau}\,d\tau$$

$$+\int_{iT_P}^{iT_P+T} K_{DR}(\tau)(1-\frac{\tau}{T}+\frac{iT_P}{T})e^{-jw\tau}\,d\tau](\frac{M-i}{M})] \quad \ldots. \quad (3.53)$$

We shall assume that over a period of T sec. $K_{DR}(\tau)$ can be assumed to be constant.

Then $$S_{n_c}(f)=2.R.P[K_{DR}(0)\int_{0}^{T}(1-\frac{\tau}{T})e^{-jw\tau}\,d\tau+\sum_{i=1}^{M-1}[\int_{iT_{P-T}}^{iT_P} K_{DR}(iT_P)$$

$$(1+\frac{\tau}{T}-\frac{iT_P}{T})e^{-jw\tau}\,d\tau+\int_{iT_P}^{iT_{P+T}} K_{DR}(iT_P)(1-\frac{\tau}{T}+\frac{iT_P}{T})$$

$$x\ e^{-jw\tau}\,d\tau]\ (\frac{M-i}{M})\ ] \quad \ldots. \quad (3.54)$$

$$= 2.R.P[\sum_{i=0}^{M-1} (\frac{M-i}{M})K_{DR}(iT_P)T(\frac{\sin\pi fT}{\pi fT})^2\cos2\pi fiT_P \quad ....(3.55)$$

$$= 2T(\frac{\sin\pi fT}{\pi fT})^2 \sum_{i=0}^{M-1} [e^{-b(iT_P)^2}\cos2\pi fiT_P + \frac{1}{4} e^{-(b+d)(iT_P)^2}$$

$$[\cos2\pi(f+f')iT_P + \cos2\pi(f-f')iT_P]] \frac{M-i}{M} \quad .... (3.56)$$

where $f'=a/2\pi i$

3.8 VALIDITY OF APPROXIMATION.

We encountered integrals of the form $\int_{iT_P}^{iT_{P+T}} R(\tau)$ $K_{DR}(\tau)e^{-jw\tau}\, d\tau$. We stated earlier that $K_{DR}(\tau)$ can be assumed to be constant over this interval. We shall verify this statement.

When the PRF=300 $T_P=1/300$

Substituting in (2.27)

$\sigma_d max = 13.2$ $\sigma_r max = V_r max = 9.9$

$a = 12x9.9 \simeq 120$

$b \simeq 8000$

$d = 2x10x169 \simeq 3400$

$$\therefore K_{DR}(\tau) = e^{-3400\tau^2} + \frac{1}{4}(e^{-11400\tau^2 - ja\tau} + e^{-11400\tau^2 + ja\tau})$$

For simplicity we shall consider only one term

Let $K'_{DR}(\tau)=e^{-3400\tau^2}$

For $\tau=iT_p+T$ $\quad K'_{DR}(iT_p+T)=e^{-3400(iT_p+T)^2}$

$$= e^{-3400[(iT_p)^2+2iTT_p]}$$

$$(\because T \ll iT_p)$$

For $\tau=iT_p$ $\quad K'_{DR}(iT_p) = e^{-3400(iT_p)^2}$

$$\therefore \frac{K'_{DR}(iT_p)}{K'_{DR}(iT_p+T)} = e^{3400(2iTT_p)}$$

This ratio and hence the variation over a period of T seconds is maximum when i is maximum. For a transmitted signal consisting of 10 pulses imax =10.

$$\therefore \left.\frac{K'_{DR}(iT_p)}{K'_{DR}(iT_p+T)}\right|_{max} = e^{3400x2x10x10^{-6}x3x10^{-3}} = e^{2x10^{-4}} = 1$$

where $T= 10^{-6}$ sec. and $T_p= \frac{1}{300}$ sec.

Next let $K_{DR}(\tau)= e^{-11400\tau^2}$

$$\text{Then } \left.\frac{K_{DR}(iT_p)}{K_{DR}(iT_p+T)}\right|_{max} = e^{6x10^{-4}} \simeq 1$$

$$aT = 120x10^{-6} \quad \therefore \frac{\cos a(iT_p+T)}{\cos aT_p i} \simeq 1$$

For $K_{DR}(\tau)=e^{-3400\tau^2}+e^{-11400\tau^2}\cos a\tau$, the variation over a period of T sec (10^{-6} sec) is negligible.

Next let us consider $\sigma_d = \frac{\sigma_d \max}{2}$, $V_{or} = \frac{V_{or}\max}{2}$, $\sigma_r = \frac{\sigma_r \max}{2}$

Then a=60 b=2x10^3 d=850

$$K_{DR}(\tau)=e^{-850\tau^2}+\tfrac{1}{2}\cos 60\tau e^{-2850\tau^2}$$

Let $K'_{DR}(\tau)= e^{-850\tau^2}$

Then $\left.\frac{K'_{DR}(iT_P)}{K'_{DR}(iT_P+T)}\right|_{max} \simeq e^{850(2iTT_P)}=e^{2.5x10^{-5}} \simeq 1$

Let $K'_{DR}(\tau) = e^{-2850\tau^2}$

$$\therefore \left.\frac{K'_{DR}(iT_P)}{K'_{DR}(iT_P+T)}\right|_{max} = e^{2850(2iTT_P)} = e^{7.5x10^{-5}} \simeq 1$$

Proceeding further in a similar fashion it can be verified that the approximation is valid for all possibles values of σ_d, σ_r and V_{or}.

3.9 THE OPTIMUM RECEIVER CORRELATION FUNCTION.

We have derived the expression for $S_{n_c}(f)$. The Fourier transform of the transmitted signal is given by

$$F_s(f) = T\left(\frac{\sin\pi fT}{\pi fT}\right) \sum_{i=0}^{M-1} e^{-jwT_P} \quad \ldots\ldots (3.57)$$

using (3.56) and (3.57)

$$G(f)= \frac{F_s(f)}{S_{n_c}(f)+No} = \frac{T(\frac{\sin\pi fT}{\pi fT}) \sum_{i=0}^{M-1} e^{-jwT_p}}{K_2(f)(\frac{\sin\pi fT}{\pi fT})^2 + No} \quad \text{.... (3.58)}$$

If we can find the inverse Fourier transform of (3.58) then the solution g(t) corresponding to the optimum receiver is known. But (3.58) is irrational and is the ratio of sinc function and hence closed form inverse function g(t) cannot be found.

3.10 PERFORMANCE EVALUATION OF THE OPTIMUM RECEIVER.

Though we could not obtain the inverse Fourier transform of (3.58) to obtain the optimum correlation waveform g(t) we can compute the performance of the optimum receiver that is possible if we had indeed used the optimum receiver correlation waveform g(t). We can compute the performance using 3.58.

If the false-alarm probability is P_F and the probability of detection is P_D then $P_F=(P_D)^{1+\Delta}$ ....(3.59)

and the performance index Δ is given by [3]

$$= \bar{E}_r \int_{T_i}^{T_f} \bar{f}_s(t)\, g^*(t)dt \quad \text{.... (3.60)}$$

where $\bar{E}_r$ is the average received energy.

Now (3.60) can be rewritten as

$$= \bar{E}_r \int_{T_i}^{T_f} \bar{f}_s(t)\ g^*(t)dt = \bar{E}_r \int_{-\infty}^{\infty} F_s(f)\ G^*(f)df \quad \ldots.(3.61)$$

using (3.58) in (3.61)

$$= \bar{E}_r \int F_s(f)\ \frac{F_s^*(f)}{No+S^*_{n_c}(f)}\ df = \bar{E}_r \int \frac{|F_s(f)|^2}{No+S_{n_c}(f)}\ df$$

If the doppler shift due to the target is f_D then

$$= \bar{E}_r \int \frac{|F_s(f-f_D)|^2}{No+S^*_{n_c}(f)}\ df \quad \ldots.\ (3.62)$$

Though closed form solution for the integral in (3.62) does not exist, Δ can be numerically evaluated. We shall consider this procedure in chapter 5.

CHAPTER-4

THE CONVENTIONAL RECEIVER

In the previous chapter we observed that the solution of the integral equation corresponding to the optimum receiver is difficult to find. However the performance of the optimum receiver can be evaluated. In light of these observations we want to consider in this chapter some structured sub optimum receivers which are relatively easy to implement.

4.1 THE CONVENTIONAL RECEIVER.

The conventional receiver is the white-noise optimum matched filter. If τ_D, f_D are the delay and doppler corresponding to the desired target the matched filter impulse response is given by

$$f_s^*(-t) = \sum_{k=0}^{M-1} f^*(-t+kT_P)e^{+jw_D t} \text{ for } \tau_D=0 \quad \ldots. (4.1)$$

$$= \sum_{k=0}^{M-1} f^*(-t+kT_P+\tau_D)e^{+jw_D t} \text{ for } \tau_D \neq 0 \quad \ldots.(4.2)$$

when the transmitted signal is $f_s'(t) = \sum_{k=0}^{M-1} f(t-kT_P)$.

The performance is degraded when the noise is coloured. The performance of the receiver is given by,

$$\Delta = \frac{E[|\ell|^2/H1] - E[|\ell|^2/H_o]}{E[|\ell|^2/H_o]} \qquad \ldots . \; (4.3)$$

where $$\ell = \int_{T_i}^{T_f} r^*(t) f_s(t) \, dt \qquad \ldots . \; (4.4)$$

In the following, we derive an expression for Δ.

For the transmitted waveform given in 3.1 i.e. equispaced rectangular M pulse train

$$\ell = \int_{T_i}^{T_f} r^*(t) \sum_{m=0}^{M-1} f(t-mT_P) e^{+jw_D t} \, dt \qquad \ldots . \; (4.5)$$

$$= \int_{0}^{T} r^*(t) f(t) e^{+jw_D t} \, dt + \int_{T_P}^{T_P+T} r^*(t) f(t-T_P) e^{+jw_D t} \, dt + \ldots .$$

$$= \int_{(M-1)T_P}^{(M-1)T_P+T} r^*(t) e^{+jw_D t} f(t-\overline{M-1}\, T_P) dt \qquad \ldots . \; (4.6)$$

Now as the doppler shift in frequency f_D due to the target is small compared to the transmitted pulse width T the phase factor $e^{-jw_D t}$ can be assumed to be constant over a period of the pulsewidth T in eqn. 4.6.

$$\ell = \int_0^T r^*(t)f(t)dt + e^{+jw_D T_P} \int_{T_P}^{T_P+T} r^*(t)f(t-T_P)dt + e^{+j(M-1)T_P w_D}$$

$$\int_{(M-1)T_P}^{(M-1)T_P+T} r\ (t)f(t-\overline{M-1}T_P)\ dt \qquad \ldots.\ (4.7)$$

substituting (3.2) in (4.7)

$$\ell = \int_0^T r^*(t)\frac{1}{\sqrt{T}}\,dt + e^{+jw_D T_P} \int_{T_P}^{T_P+T} r^*(t)\frac{1}{\sqrt{T}}\,dt + e^{j\overline{M-1}T_P w_D}$$

$$\int_{(M-1)T_P}^{(M-1)T_P+T} r^*(t).\ \frac{1}{\sqrt{T}}\ dt \qquad \ldots.\ (4.8)$$

$$\text{Let } c_i = e^{jw_D(iT_P)} \int_{iT_P}^{iT_P+T} r^*(t)dt \qquad \ldots.\ (4.9a)$$

$$\text{Then } \ell = \frac{1}{\sqrt{T}}(c_o+c_1+c_2\ldots\ c_{M-1}) = [c_o\ c_1\ldots\ c_{M-1}] \times \begin{bmatrix} \frac{1}{\sqrt{T}} \\ \frac{1}{\sqrt{T}} \\ \cdot \\ \cdot \\ \cdot \\ \frac{1}{\sqrt{T}} \end{bmatrix}_{M\times 1}$$

i.e. $\ell \triangleq CU$ where $C=[c_o\ c_1\ldots c_{M-1}]$

$$U = \left[\ \frac{1}{\sqrt{T}}\ \ldots.\ \frac{1}{\sqrt{T}}\ \right]^T_{1\times M} \qquad \ldots.(4.9b)$$

$$\therefore \|\ell\|^2 = \ell^{*T}\ell = U^{*T}C^{*T}CU = U^T C^{*T}CU$$

$$\therefore E[\|\ell\|^2] = U^T E[C^{*T}C]U = U^T kU \quad \ldots. (4.10)$$

where $k(i,j) = E[c_i^* c_j]$

Now

$$E[c_i^* c_j] = E[\int_{jT_p}^{jT_p+T} \int_{iT_p}^{iT_p+T} r^*(t) r(u) dt du \gamma^{j-i} \quad \ldots. (4.11)$$

where $\gamma = e^{\sqrt{-1}\, w_D T_P}$

Eqn. (4.11) can be rewritten as (using 2.21)

$$E[c_i^* c_j] = E[\int_{jT_P}^{jT_P+T} \int_{iT_P}^{iT_P+T} [B^* f^*(t-iT_P)\gamma^{*i} + n_c^*(t) + w^*(t)]$$

$$[Bf(u-jT_P)\gamma^j + n_c(u) + w(u)] dt du \gamma^{j-i}] \quad \ldots. (4.12)$$

$$= [\iint [E[|B|^2 f^*(t-iT_P)\gamma^{*i} f(u-jT_P)] + E[B^* f^*(t-iT_P)\gamma^{*i} n_c(u)]$$

$$+ E[B^* f^*(t-iT_P)\gamma^{*i} w(u)] + E[Bf(u-jT_P)\gamma^j n_c^*(t)] + E[n_c^*(t) n_c(u)]$$

$$+ E[n_c^*(t) w(u)] + E[w^*(t)\gamma^j f(u-jT_P)B] + E[w^*(t) n_c(u)]$$

$$+ E[w^*(t) w(u)] \; dt du] \, \gamma^{j-i} \quad \ldots. (4.13)$$

If we assume that the clutter return and receiver white noise are uncorrelated and assume that

$$E[w^*(t)w(u)] = No\delta(t-u)$$
$$E[n_c^*(t)n_c(u)] = K_{n_c}(t,u) \qquad \ldots\ldots (4.14)$$

Then (4.13) becomes

$$E[c_i^* c_j] = \gamma^{-i+j}\iint [K_{n_c}(t,u) + No\delta(t-u) + f^*(t-iT_p)f(u-jT_p)\gamma^{-j+i}$$
$$E[|B|^2]dtdu \qquad \ldots\ldots (4.15)$$

Now $$\int_{jT_p}^{jT_p+T}\int_{iT_p}^{iT_p+T} No\delta(t-u)dtdu = No\delta(i-j)T \qquad \ldots\ldots (4.16a)$$

and $$\int_{jT_p}^{jT_p+T}\int_{iT_p}^{iT_p+T} f^*(t-iT_p)f(u-jT_p)\gamma^{-j+i}E[|B|^2]dtdu$$

$$= \int_{jT_p}^{jT_p+T} f(u-jT_p)\int_{iT_p}^{iT_p+T} f^*(t-iT_p)\gamma^{-j+i}dtdu\ E[|B|^2] \qquad \ldots\ldots (4.16b)$$

$$= \frac{1}{\sqrt{T}}\cdot\frac{1}{\sqrt{T}}\cdot T\cdot T\cdot\gamma^{j-i}E[|B|^2] = \gamma^{-j+i}TE[|B|^2] \qquad \ldots\ldots (4.17)$$

Now $$\int_{jT_P}^{jT_P+T} \int_{iT_P}^{iT_P+T} K_{n_c}(t,u)dtdu = \iint K_{DR}(t-u)R(t-u)dtdu$$

.... (4.18)

where R(t-u) is given by (3.3) and is reproduced here for convenience

$$R(t-u) = \sum_{k=0}^{M-1} \frac{M-k}{M}[1-\frac{\beta(\tau k)}{T}] \qquad \text{.... (4.19)}$$

where $\beta(\tau_k) = |\tau-kT_P|$ for $|\tau-kT_P| \leqslant T$

$= 0$ otherwise

Since $K_{DR}(t-u)$ is the lowpass clutter process it is slowly varying and hence over a short period of time T=pulsewidth it can be assumed to be constant. We verified this in the last chapter.

Then

$$\int_{jT_P}^{jT_P+T} \int_{iT_P}^{iT_P+T} K_{DR}(t-u)R(t-u)dtdu = \int_{jT_P}^{jT_P+T} K_{DR}(iT_P-T) \int_{iT_P}^{iT_P+T} R(t-u)dtdu$$

$$= K_{DR}(iT_P-jT_P) \int_{jT_P}^{jT_P+T} \int_{iT_P}^{iT_P+T} R(t-u)dtdu$$

.... (4.20)

Let $A(k) = \frac{M-k}{MT}$ .... (4.21)

To evaluate $\iint R(t-u)dtdu$ (i.e.)(4.20) we shall consider the various values of t,u and the corresponding values of k for which $|\tau-kT_p| \leq T$ and hence we shall find R(t-u). The values are given in table 4.1

$$k= -[(t-u)\bmod T_p]+(t-u) \quad .... (4.22)$$

No	t	u	t-u	k	R(t-u)	$\tau-kT_p$
1	iT_p	jT_p	$(i-j)T_p$	$(i-j)$	$A(i-j)[1-\frac{0}{T}]$	0
2	$iT_p+\frac{1}{2}T$	''	$(i-j)T_p+\frac{T}{2}$	''	$A(i-j)[1-\frac{T}{2T}]$	$\frac{T}{2}$
3	iT_p+T	''	$(i-j)T_p+T$	''	$A(i-j)[1-\frac{T}{T}]$	T
4	iT_p	$jT_p+\frac{T}{3}$	$(i-j)T_p-\frac{T}{3}$	$(i-j)$	$A(i-j)[1-\frac{T}{3T}]$	$\frac{T}{3}$
5	$iT_p+\frac{T}{3}$	''	$(i-j)T_p$	''	$A(i-j)[1-\frac{0}{T}]$	0
6	$iT_p+\frac{2}{3}T$	''	$(i-j)T_p+\frac{T}{3}$	''	$A(i-j)[1-\frac{T}{3T}]$	$\frac{T}{3}$
7	iT_p+T	''	$(i-j)T_p+\frac{2}{3}T$	''	$A(i-j)[1-\frac{2}{3}\frac{T}{T}]$	$\frac{2}{3}T$
8	iT_p	$jT_p+\frac{T}{2}$	$(i-j)T_p-\frac{T}{2}$	$(i-j)$	$A(i-j)[1-\frac{T}{2T}]$	$\frac{T}{2}$
9	$iT_p+T/2$	''	$(i-j)T_p$	''	$A(i-j)[1-\frac{0}{T}]$	0
10	iT_p+T	''	$(i-j)T_p+\frac{T}{2}$	''	$A(i-j)[1-\frac{T}{2T}]$	T/2

TABLE 4.1

From 1,2,3 entries in table 4.1 we infer that

$$\int_{iT_P}^{iT_P+T} R(t-jT_P)dt = A(i-j)\int_0^T \left(1-\frac{x}{T}\right)dx \quad \ldots\ (4.23)$$

From the entries numbered 4,5,6,7 we infer that

$$\int_{iT_P}^{iT_P+T} R(t-jT_P-\tfrac{T}{3})dt = \int_{-\frac{2}{3}T}^{0} \left(1+\frac{x}{T}\right)dx + \int_0^{\frac{1}{3}T}\left(1-\frac{x}{T}\right)dx \quad \ldots\ (4.24)$$

From the entries numbered 8,9,10 we get

$$\int_{iT_P}^{iT_P+T} R(t-jT_P-\tfrac{T}{2})dt = A(i-j)\left[\int_{-\frac{T}{2}}^{0} \left(1+\frac{x}{T}\right)dx + \int_0^{T/2}\left(1-\frac{x}{T}\right)dx\right] \quad \ldots\ (4.25)$$

combining (4.23), (4.24) and (4.25) we get

$$\int_{iT_P}^{iT_P+T} R(t-jT_P-y)dt = A(i-j)\left[\int_{-(T-y)}^{0} \left(1+\frac{x}{T}\right)dx + \int_0^{y}\left(1-\frac{x}{T}\right)dx\right] \quad \ldots (4.26)$$

$$\therefore \int_{jT_P}^{jT_P+T}\int_{iT_P}^{iT_P+T} R(t-u)dtdu = \int_0^{T}\int_{iT_P}^{iT_P+T} R(t-jT_P-y)dtdy$$

$$= \int_0^T A(i-j)\left[\int_{-(T-y)}^{0} \left(1+\frac{x}{T}\right)dx + \int_0^{y}\left(1-\frac{x}{T}\right)dx\right]dy \quad \ldots\ (4.27)$$

$$= A(i-j)\,\frac{2}{3}T^2 \qquad \ldots\ (4.28)$$

substituting (4.28) in (4.20) we get

$$\int_{jT_P}^{jT_P+T}\int_{iT_P}^{iT_P T} K_{n_c}(t,u)\,dt\,du = K_{DR}(iT_P-jT_P)\left[\frac{M-(i-j)}{M}\right]\frac{2}{3}T \qquad \ldots\ (4.29)$$

Substituting (4.16a),(4.17) and (4.29) in (4.15) we get

$$E[c_i^* c_j] = No\,\delta(i-j)T + \frac{2}{3}T\left[\frac{M-(i-j)}{M}\right]K_{DR}(i-j)T_P\gamma^{(i-j)} + TE[|B|^2] \qquad \ldots\ (4.30)$$

Let k_1 be a matrix whose elements are

$$k_1(i,j) = E[c_i c_j] \qquad \ldots\ (4.31)$$

$$\therefore\ E[|\ell|^2/H_1] = U^T k_1 U \qquad \ldots\ (4.32)$$

Similarly $E[|\ell|^2/H_o] = U^T k_o U$ .... (4.33)

where k_o is an MxM matrix whose (i,j)th elements are given by

$$K_o(i,j) = K_1(i,j) - TE[\ B\ ^2] \qquad \ldots\ (4.34)$$

$$\therefore\ \Delta = \frac{U^T K_2 U}{U^T K_o U} \qquad \ldots\ (4.35)$$

where K_2 is an MxM matrix whose (i,j) the element is given by

$$K_2(i,j) = TE[|B|^2],\quad i,j = 0,1,2\ldots M-1 \qquad \ldots(4.35a)$$

The Vector U is defined in eqn. 4.9. The expression for the performance index Δ derived in [14] also has the same form as (4.35) excepting that the matrices K_2, K_0 are defined and obtained using different statistics.

AN EXAMPLE

The computation of the performance index can be made clear by considering a simple example. Let the total number of transmitted pulses be three. Then the elements of the relevant matrices are

$$K_0(0,0)=N_oT+\frac{2}{3}TK_{DR}(0) \qquad \text{Let } E[|B|^2]=1$$

$$K_0(0,1)=\frac{2}{3}T\left(\frac{3-1}{3}\right)K_{DR}(1)\gamma^{-1}=\frac{4}{9}\,T\,K_{DR}(1)$$

$$K_0(2,2)=N_oT+\frac{2}{3}TK_{DR}(0)$$

$$\therefore\ U^TK_2U=\left[\frac{1}{\sqrt{T}}\ \frac{1}{\sqrt{T}}\ \frac{1}{\sqrt{T}}\right]\begin{bmatrix} T & T & T \\ T & T & T \\ T & T & T \end{bmatrix}\begin{bmatrix} \frac{1}{\sqrt{T}} \\ \frac{1}{\sqrt{T}} \\ \frac{1}{\sqrt{T}} \end{bmatrix}=9 \quad \ldots.(4.36)$$

$$U^T K_O U = \left[\frac{1}{\sqrt{T}}\ \frac{1}{\sqrt{T}}\ \frac{1}{\sqrt{T}}\right] \begin{bmatrix} NoT+\frac{2}{3}TK_{DR}(0) & \frac{4}{9}TK_{DR}(1)\gamma^{-1} & \frac{2}{9}K_{DR}(2)\gamma^{-2} \\ \frac{4}{9}TK_{DR}(1)\gamma & NoT+\frac{2}{3}TK_{DR}(0) & \frac{4}{9}TK_{DR}(1)\gamma^{-1} \\ \frac{2}{9}TK_{DR}(2)\gamma^{2} & \frac{4}{9}TK_{DR}(1)\gamma & NoT+\frac{2}{3}TK_{DR}(0) \end{bmatrix}$$

where $\gamma = e^{\sqrt{-1}w_D T_P}$

$$\times \begin{bmatrix} \frac{1}{\sqrt{T}} \\ \frac{1}{\sqrt{T}} \\ \frac{1}{\sqrt{T}} \end{bmatrix}$$

$$= 3No + \frac{2}{3}K_{DR}(0) + \frac{16}{9}K_{DR}(1)\cos w_D T_P + \frac{8}{9}K_{DR}(2)\cos w_D(2T_P) \quad \ldots.(4.37)$$

substituting (4.36) and (4.37) into (4.35) we get .

From (4.37) we observe that Δ is minimum when $w_D=0$. Hence maximum degradation in performance occurs when the target doppler shift =0. As the doppler shift increases Δ also increases. In the next chapter we compare the performance of the conventional receiver with that of the optimum receiver.

4.2 MISMATCHED RECEIVER.

The conventional receiver is simple to implement. But the performance of the conventional receiver deteriorates as the clutter/white noise ratio increases and when the relative velocity between the target and clutter is zero. The mismatched receiver is the receiver which retains the simplicity

of the conventional receiver but performs better than the conventional receiver when chosen appropriately.

. For a transmitted signal of the form $f_s'(t)=\sum_{k=o}^{M-1} f(t-KT_p)$ the mismatched receiver uses a correlation waveform consisting of constant amplitude (rectangular) uniformly spaced pulses with different weights V_k'S.

If the doppler shift due to the target is f_D the receiver correlation waveform

$$V(t)=\sum_{k=o}^{M-1} v_k f(t-kT_p)e^{-jw_D t} \qquad \text{.... (4.38a)}$$

The optimum v_k's which maximizes the performance of the receiver will be found in the next sections.

4.3 PERFORMANCE INDEX OF THE MISMATCHED RECEIVER.

The mismatched receiver computes

$$\ell_m=\int r(t)\,v^*(t)dt \qquad \text{.... (4.38b)}$$

and compares $|\ell_m|^2$ with a threshold. v(t) is the arbitrary function that we want to choose [5]

The performance of the receiver is specified by Δm where

$$\Delta m = \frac{E[|\ell_m|^2/H_1]-E[|\ell_m|^2/H_0]}{E[|\ell_m|^2/H_0]} \qquad \text{.... (4.39)}$$

using (2.21) and (2.23) in (4.38b)

$$E[|\ell_m|^2/H_1]=E[\,|\int_{T_i}^{T_f}[Bf_s(t)+n(t)]v^*(t)dt\,|^2\,]$$

$$=\iint K_n(t,u)v^*(t)v(u)dtdu+\iint E[Bf_s(t)n^*(u)]v^*(t)v(u)dtdu$$

$$+\iint E[B^*f_s^*(u)n^*(t)]v^*(t)v(u)dtdu+\iint E[|B|^2]f_s(t)f_s^*(u)$$

$$v^*(t)v(u)dtdu$$

If the signal and noise are uncorrelated

Then

$$E[|\ell_m|^2/H_1] =\iint K_n(t,u)v^*(t)v(u)dtdu+\iint f_s(t)E[|B|^2]f_s^*(u)v(t)v(u)dtdu \qquad \ldots.(4.40)$$

Similarly $E[|\ell_m|^2/H_0)=\iint v^*(t)v(u)K_n(t,u)dtdu \qquad \ldots.(4.41)$

Substituting (4.40) and (4.41) in (4.39) we get

$$\Delta_m = \frac{E[|B|^2]|\int f_s(t)v^*(t)dt|^2]}{\iint v^*(t)v(u)K_n(t,u)dtdu} \qquad \ldots.(4.42)$$

For the uniform doppler profile case

$$K_n(t,u)=\int_{-\infty}^{\infty}K_{DR}(t-u)f_s(t-\mu)f_s(u-\mu)d\mu +No\delta(t-u)$$

$$= K_{DR}(t-u)\int_{-\infty}^{\infty} f_s(t-\mu)f_s(u-\mu)d\mu +No\delta(t-u) \qquad \ldots.(4.43)$$

Substituting (4.43) in (4.42)

$$\Delta_m = \frac{E[|B|^2]|\int f_s(t)v^*(t)dt|^2}{No+\iiint v^*(t)v(u)K_{DR}(t-u)f_s(t-\mu)f_s(u-\mu)dtdud\mu}$$

$$\ldots.(4.44)$$

.4 MAXIMIZATION OF THE PERFORMANCE INDEX FOR MISMATCHED RECEIVER

For maximizing Δ_m we can maximize $\left|\int f_s(t)v^*(t)dt\right|^2$ eeping the denominator of Δ_m constant.

.e Maximize $\left|\int f_s(t)v^*(t)dt\right|^2$ subject to the constraint

$$\iint v^*(t)v(u)K_n(t,u)dtdu = C \qquad \text{.... (4.45)}$$

sing the Lagrange multiplier λ we shall form the function L hich has to be maximized without any constraint.

.e $L = \left|\int f_s(t)v^*(t)dt\right|^2 + \lambda\left[\iint v^*(t)v(u)K_n(t,u)dtdu - C\right]$

$$\text{.... (4.46)}$$

ow (4.49) can be rewritten as

$$= \iint f_s(t)f_s(u)v^*(t)v(u)dtdu + \lambda\left[C - \iint v\ (t)v(u)K_n(t,u)dtdu\ \right]$$

$$\text{.... (4.47)}$$

e want to maximize L by choosing v(t). Let $v_o(t)$ be the unction which maximizes L w.r.t. the integration w.r.t the ariable t. Let us consider the effect of variation of $v_o(t)$ y $\epsilon h(t)$. With

$$v(t) = v_o(t) + \epsilon h(t) \qquad \text{.... (4.48)}$$

et the value of L be denoted as L_ϵ.

hen

$$L_\epsilon \triangleq \iint f_s(t)f_s(u)[v_o(t)+\epsilon h^*(t)]v(u)dtdu$$

$$+ \quad [C - \iint [v_o(t)+\epsilon h^*(t)]K_n(t,u)v(u)dtdu \qquad \text{.... (4.49)}$$

f $v_o(t)$ is indeed the optimum v(t) which maximizes w.r.t t then the variational term in L_ϵ should be zero [10]

i.e $\iint f_s(t)f_s(u)\in h^*(t)v(u)dudt - \iint h^*(t)K_n(t,u)v(u)dtdu = 0$

.... (4.50)

(i.e) $\iint h^*(t)\in[\ f_s(t)f_s^*(u)v(u) - \ K_n(t,u)v(u)]dtdu = 0$

.... (4.51)

If this is to be true for any arbitrary $h^*(t)$ then it will be possible only when

$$f_s(t)\int f_s^*(u)v(u)du - \lambda\int K_n(t,u)v(u)du = 0$$

i.e $f_s(t)C_1 = \lambda\int K_n(t,u)v(u)du$

$$\therefore\ f_s(t) = \frac{\lambda}{C_1}\int K_n(t,u)v(u)du = \lambda_1\int K_n(t,u)v(u)du \quad \text{....(4.52)}$$

where $C_1 = \int f_s^*(u)v(u)du \qquad \lambda_1 = \frac{\lambda}{C_1}$ .... (4.53)

Similarly it can be shown that the optimum v(u)which maximizes L w.r.t integration w.r.t u also satisfies the same equation. ∴ L is maximum when v(t) and v(u) satisfies the equation (4.53).

When we maximize Δ_m with no contraint on v(t) the optimum v(t) is the same as the optimum correlation function g(t) that we shall use for a coloured noise. However when we restrict the v(t) to be of the form given in equation (4.38a).the solution of 4.53 for this v(t) yields the optimum values for the weights.

4.5 DETERMINATION OF WEIGHT VECTOR.

The correlation waveform v(t) corresponding to a mismatched receiver given by (4.38a) can be rewritten in vector form as

$$v(t) = [V_o, V_1\ V_2 \ldots V_{M-1}] \begin{bmatrix} f(t) \\ f(t-T_P)\gamma \\ \cdot \\ \cdot \\ \cdot \\ f(t-mT_P)\gamma^{M-1} \end{bmatrix}$$

where $\gamma = e^{-jw_D T_P}$

From (4.53) the optimum values of v_i $i=o, 1.. M-1$ can be found as follows :

substituting (4.38a) and (4.1) in (4.53).

$$\sum_{k=o}^{M-1} f(t-kT_P)e^{jw_D t} = \int_{T_i}^{T_f} K_n(t,u) \sum_{k=o}^{M-1} v_k f(u-kT_P)e^{jw_D u}\, du$$

$$= \int_0^T K_n(t,u)v_o f(u)du + \int_{T_P}^{T_P+T} K_n(t,u)v_1 f(u-T_P)e^{jw_D T_P u}\, du + \ldots$$

$$+ \int_{(M-1)T_P}^{(M-1)T_P+T} K_n(t,u)V_{M-1} f(u-\overline{M-1}T_P)e^{jw_D (M-1)T_P u}\, du \quad \ldots.(4.54)$$

$$= \frac{1}{\sqrt{T}} [C_O(t)\ \ C_1(t)\ \ldots.C_{M-1}(t)] \begin{bmatrix} V_O \\ V_1\gamma \\ V_2\gamma^2 \\ \cdot \\ \cdot \\ \cdot \\ V_{M-1}\gamma^{M-1} \end{bmatrix} \quad \ldots.(4.55)$$

where $\gamma = e^{jw_D T_P}$

$$C_i(t) = \int_{iT_P}^{iT_P+T} K_n(t,u)\,du \qquad \text{.... (4.56)}$$

Let $t = \ell T_P$. $\ell = 0, 1 .. M-1$

$$\therefore C_i(\ell T_P) = \int_{iT_P}^{iT_P+T} K_n(t,u)du = \int_{iT_P}^{iT_P+T} K_{n_c}(t,u)du + No\delta(t-u)$$

$$= \int_{\ell T_P}^{\ell T_P+T} K_{n_c}(\ell T_P - u)du + No\delta(i-\ell)$$

$$= K_{DR}(\ell T_P - iT_P) \int_{iT_P}^{iT_P+T} R(\ell T_P - u)du + No\delta(i-\ell) \qquad \text{....(4.56)}$$

($\because$ $K_{DR}(t-u)$ is slowly varying)

Now $$\int_{iT_P}^{iT_P+T} R(\ell T_P - u)du = A(i-\ell)\int_0^T (1-\frac{x}{T})dx = \frac{T}{2}A(i-\ell)$$

$$= \frac{M-(i-\ell)}{2M} \qquad \text{.... (4.57)}$$

Substituting (4.57) in (4.56) we get

$$C_i(\ell T_P) = K_{DR}(\ell - i)T_P \cdot \frac{M-\overline{i-\ell}}{2M} + No\delta(i-\ell) \qquad \text{.... (4.58)}$$

$$\therefore f(0) = \frac{1}{\sqrt{T}}[C_0(0)\ C_1(0)\ \ldots .\ C_{M-1}(0)]\begin{bmatrix} V_0 \\ V_1\gamma \\ \cdot \\ \cdot \\ \cdot \\ V_{M-1}\gamma^{M-1} \end{bmatrix} \qquad \text{.... (4.59)}$$

However $f(0) = \frac{1}{\sqrt{T}}$

Similarly

$$f(T_P) = \frac{\gamma}{\sqrt{T}} = \frac{1}{\sqrt{T}}\,[C_0(T_P)\;C_1(T_P)\;\ldots\;C_{M-1}(T_P)]\begin{bmatrix} V_0 \\ V_1\gamma \\ \cdot \\ \cdot \\ \cdot \\ V_{M-1}\gamma^{M-1}\end{bmatrix} \quad (4.60)$$

and

$$f(\overline{M-1}T_P) = \frac{\gamma^{M-1}}{\sqrt{T}}\,[C_0(\overline{M-1}T_P)\,C_1(\overline{M-1}T_P)\;\ldots\;C_{M-1}(\overline{M-1}\;T_P)]\;\mathbf{V} \quad \ldots\ (4.61)$$

From (4.59), (4.60) and (4.61) we infer the following relation

$$\begin{bmatrix} 1 \\ \gamma \\ \gamma^2 \\ \cdot \\ \cdot \\ \cdot \\ \gamma^{M-1}\end{bmatrix} = \begin{bmatrix} C_0(0) & \ldots\ldots & C_{M-1}(0) \\ C_0(T_P) & \ldots\ldots & C_{M-1}(T_P) \\ \cdot & & \\ \cdot & & \\ \cdot & & \\ \cdot & & \\ C_0(M-1T_P) & \ldots & C_{M-1}(\overline{M-1}T_P)\end{bmatrix}\begin{bmatrix} V_0 \\ V_1\gamma \\ V_2\gamma^2 \\ \cdot \\ \cdot \\ \cdot \\ V_{M-1}\gamma^{M-1}\end{bmatrix} \quad \ldots (4.62)$$

$$= \quad [\mathcal{E}] \begin{bmatrix} 1 & & & 0 \\ & \gamma & & \\ & & \gamma^2 & \\ 0 & & \ddots & \\ 0 & & & \gamma^{M-1}\end{bmatrix}\begin{bmatrix} V_0 \\ V_1 \\ V_2 \\ \vdots \\ V_{M-1}\end{bmatrix} \quad \ldots (4.63)$$

i.e $F = \mathcal{E}\,\Gamma^{-1}V$ where $\Gamma = \begin{bmatrix} 1 & & 0 \\ & \gamma & \\ & & \cdot \\ 0 & & \cdot \\ 0 & & \gamma^{M-1}\end{bmatrix}$ and $V = \begin{bmatrix} V_0 \\ \cdot \\ \cdot \\ \cdot \\ V_{M-1}\end{bmatrix}$

$$F = [\,1\;\gamma\;\gamma^2\ldots\gamma^{M-1}]^T \quad \ldots\ (4.64)$$

$$V = \Gamma^{-1}\mathcal{E}^{-1} F \qquad \therefore V = \Gamma^{-1}\mathcal{E}^{-1} F \qquad \ldots\ldots (4.64a)$$

The weight vector for the mismatched receiver has been obtained by Rummler[20][21], Spafford et al [22] using different methods for maximation. The final result obtained is the same as that we have obtained here.

AN EXAMPLE

Let us consider a transmitted waveform consisting of 3 rectangular pulses

Then

$$\Gamma = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \gamma & 0 \\ 0 & 0 & \gamma^2 \end{bmatrix} ; F = \begin{bmatrix} 1 \\ \gamma \\ \gamma^2 \end{bmatrix} \qquad \ldots\ldots (4.65)$$

$$E(0,0) = C_0(0) = No + \frac{K_{DR}(0)}{2} = E(2,2) = E(1,1)$$

$$E(0,1) = C_0(T_P) = K_{DR}(T_P)\tfrac{1}{2}\,\tfrac{2}{3} = E(1,0) \qquad \ldots\ldots (4.66)$$

$$E(0,2) = C_0(2T_P) = K_{DR}(2T_P)\cdot\tfrac{1}{2}\cdot\tfrac{1}{3} = E(2,0)$$

$$E(2,1) = \frac{K_{DR}(T_P)}{2}\cdot\frac{2}{3} = E(1,2)$$

$$\therefore \begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \gamma^{-1} & 0 \\ 0 & 0 & \gamma^{-2} \end{bmatrix} \begin{bmatrix} No + \frac{K_{DR}(0)}{2} & \frac{K_{DR}(T_P)}{3} & \frac{K_{DR}(T_P)}{6} \\ \frac{K_{DR}(T_P)}{3} & No + \frac{K_{DR}(0)}{2} & \frac{K_{DR}(T_P)}{3} \\ \frac{K_{DR}(T_P)}{6} & \frac{K_{DR}(T_P)}{3} & No + \frac{K_{DR}(0)}{2} \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ \gamma \\ \gamma^2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \gamma^{-1} & 0 \\ 0 & 0 & \gamma^{-2} \end{bmatrix} \begin{bmatrix} g_1 & g_2 & g_3 \\ g_2 & g_1 & g_2 \\ g_3 & g_2 & g_1 \end{bmatrix} \begin{bmatrix} 1 \\ \gamma \\ \gamma^2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \gamma^{-1} & 0 \\ 0 & 0 & \gamma^{-2} \end{bmatrix} \begin{bmatrix} g_1+g_2\gamma+g_3\gamma^2 \\ g_2+g_1\gamma+g_2\gamma^2 \\ g_3+g_2\gamma+g_1\gamma^2 \end{bmatrix} = \begin{bmatrix} g_1+g_2\gamma+g_3\gamma^2 \\ g_2\gamma^{-1}+g_1+g_2\gamma \\ g_3\gamma^{-2}+g_2+g_1\gamma \end{bmatrix} \quad \ldots.(4.67)$$

$\therefore$ V_0, V_1, V_2 are complex numbers.

4.5 PERFORMANCE EVALUATION OF MISMATCHED RECEIVER.

Proceeding as in the conventional receiver we can derive an expression for Δ_m the performance index and

$$\Delta m = \frac{V^{*T} K_2 V}{V^{*T} K_0 V} \quad \ldots. (4.68)$$

where K_2, K_0 are NxM matrices whose elements are defined in equations (4.31),(4.34) and (4.35a). It can be verified that Δ_m is real though V is complex. The performance of the mismatched receiver is numerically evaluated in chapter 5 using equation 4.68.

4.6 ADAPTIVE MISMATCHED RECEIVER.

The performance of the mismatched receiver is evidently non-inferior to that of the conventional receiver.

This is because even when the noise is predominantly white the mismatched receiver will adjust its coefficients such that it is nothing but a conventional receiver and hence it is optimum. As the noise becomes more and more coloured the mismatched filter coefficients change to combat the effect of noise (clutter) though not completly. For a conventional receiver the knowledge of the clutter characteristics is of no advantage. But a mismatched receiver filter coefficients can be adjusted with varying clutter characteristics to obtain better performance. Thus even in varying clutter environment the performance of mismatched receiver can be made superior compared to conventional receiver.

The characterization of the clutter statistics by equation (3.6) is handy. The moment we estimate the parameters $V_{or}, \sigma_r, \sigma_d, V_{od}$ the clutter characteristics are known. So the mismatched receiver coefficients can be computed. In a changing clutter environment it is enough to change the mismatched receiver coefficients only when the clutter characteristics change significantly. Or in other words it is necessary to compute new set of filter coefficients only when the parameters change significantly. In this connection the parameters are quantized and the measured clutter characteristics is classified to that quantized parameter set to which it is the nearest. For determining the quantization step for the parameters

$V_{or}, V_{od}, \sigma_r, \sigma_d$ we specified the maximum degradation in performance due to quantization as 0.1 %. By trial and error we arrived at the quantization step that will result in a performance degradation less than or equal to 0.1% . The computer program output at some stage of the trial and error procedure is given in table 5.3 It is found that the parameters $(\sigma_r, \sigma_d, V_{or} V_{od})$ require (2,8,2,8) quantization levels respectively.

4.7 ESTIMATION OF THE CLUTTER PARAMETERS.

The first step in the adaptive receiver is the estimation of the parameters. The parameters $\sigma_r, V_{or}, \sigma_d, V_{od}$ are to be estimated from the covariance function of noise. i.e. the covariance function of the received signal when the target is absent. The covariance function of the noise

$$K_n(t,u) = K_{n_c}(t,u) + No\delta(t-u)$$

$K_{n_c}(t-u)$ is given by (2.17) which we reproduce here for convenience

$$K_{n_c}(t-u) = \frac{2}{3} K_1(1+\cos a\tau e^{-b\tau^2}) e^{jc\tau - d\tau^2} R(\tau) \quad \ldots. (4.71)$$

where

$$\begin{aligned} a &= 2V_r \times 2\pi \\ b &= 2\sigma_r^2 \times 4\pi^2 \\ c &= 2\pi V_d \\ d &= \tfrac{1}{2}\sigma_d^2 \times 4\pi^2 \end{aligned} \quad \ldots. (4.72)$$

and $R(\tau)$ is given by equation (3.3) and (3.4) which is reproduced here for convenience.

$$R(\tau) = \sum_{k=0}^{M-1} \frac{M-K}{M} \left(1 - \frac{\beta(\tau_k)}{T}\right) \quad \ldots\ldots (4.73)$$

$$\text{where } \beta(\tau_k) = \tau - KT_P \quad \text{if} \quad |\tau - KT_P| < T \quad \ldots\ldots (4.73)$$
$$= 0 \quad \text{otherwise}$$

We observe that when $\tau = NT_P$

$$\text{then } R(\tau) = \frac{M-N}{M} \quad \ldots\ldots (4.74)$$

$$\text{Then } K_{n_c}(NT_P) = \frac{2}{3} K_1\left(\frac{M-N}{M}\right)(1+\cos a\tau e^{-b\tau^2})e^{jc\tau - d\tau^2} \quad \ldots\ldots (4.75)$$

where $\tau = NT_P$

From the observed data the covariance function of the noise can be computed [27] and for delays $\tau = NT_P$; $N = 0, 1..M-1$ its value is given by eqn. 4.75. Using this $K_{n_c}(\tau)$ the parameters a, b, c, d are to be estimated. Since the unknown parameters are exponentially related to $K_{n_c}(\tau)$ it is difficult to estimate the parameters directly.

Taking logarithm of (4.75)

$$\log K_{n_c}(NT_P) = K_2 + \log(M-N) + \log(1+\cos a\tau e^{-b\tau^2}) + jc\tau - d\tau^2 \quad \ldots\ldots (4.76)$$

For N=

$$\log K_{n_c}(T_P) = K_2 + \log(M-1) + \log(1+\cos aT_P e^{-bT_P^2}) + jcT_P - dT_P^2 \quad \ldots (4.77)$$

For N=2

$$\log K_{n_c}(2T_P)=K_2+\log(M-2)+\log(1+\cos 2aT_P e^{-4bT_P^2})+jcT_P-dT_P^2 \quad \ldots.(4.78)$$

Since $\cos aT_P e^{-bT_P^2} \angle 1$ $\log(1+\cos aT_P e^{-bT_P^2})$ can be expanded in power series. For simplicity we shall retain only 2 terms in the expansion then (4.77) becomes

$$\log K_{n_c}(T_P)=K_2+\log(M-1)+xy-\frac{x^2y^2}{2}+jcT_P-dT_P^2 \quad \ldots. (4.79)$$

$$\log K_{n_c}(2T_P)=K_2+\log(M-2)+\cos 2aT_P e^{-4bT_P^2}-\frac{\cos 2aT_P e^{-4bT_P^2}}{2}$$
$$+ 2jcT_P-4dT_P^2$$

$$= K_2+\log(M-2)+(2x^2-1)y^4-\frac{(2x^2-1)^2y^8}{2}+2jcT_P+4dT_P \quad \ldots..(4.80)$$

where $x= \cos aT_P$ $y=e^{-bT_P^2}$

combining (4.79) and (4.80) we can get a multinomial equation in x,y eliminating the parameters c and d. However this procedure does not make use of the complete observed data and the solution for the multinomial equation is difficult to obtain.

However we know from last section that the parameters $(\sigma_r, V_{or}, \sigma_d, V_{od})$ take only (2,2,8,8) quantized levels. Hence (a,b,c,d) take only 256 combinations. Our job is to find which parameter set closely fits the observed data. From

(4.75) we observe that the parameter c can be evaluated by taking the ratio of the real part of $K_{n_c}(\tau)$ by the imaginary part. Having found the value of c we are left with only 32 combinations. We have to decide to which parameter set of these 32 combinations the measured data is the closest. For this we can adopt the following procedure :

Let us find the covariance function of the received signal when the target is absent. Let the value of this for delays equal 0, T_P... (M-1) T_P be arranged in a vector. Similarly using (4.75) for each possible parameter set covariance vector would be found. Then the problem becomes one of finding the nearest vector [12] to the observed vector from the 32 vectors. By finding the distance we can find the nearest vector.

4.8 ADAPTIVE RADAR RECEIVER SCHEME.

We shall summarize the above sections. The clutter parameters are estimated from the observed data as in the last section. The mismatched filter coefficients are precomputed and stored for each parameter set. Hence once the parameters are known the optimum mismatched Receiver filter coefficients are read. Since this scheme requires very little computational efforts it can adapt fast to thechanging clutter environment.

4.9 DISCRETE RESOLUTION RECEIVER.

The next receiver that is easy to implement that we consider is the discrete resolution receiver. The performance

we have replaced the dummy parameters m,k by m'k' in equation (4.84)

(4.84) can be rewritten as

$$\int \sum_{m=o}^{M-1} f(t-k'T_o-m'T_P)f^*(t-mT_P)e^{-jw_Dt}\,dt$$

$$+\int \sum_{m=o}^{M-1}\sum_{k=o}^{N-1} f^*(t-kT_o-mT_P)f(t-k'T_o-m'T_P)g^*_{mk}\,dt=0 \quad \ldots.\ (4.85)$$

For a rectangular pulse defined by (3.1)

$$f(t-kT_o-mT_P)f^*(t-k'T_o-m'T_P)=\frac{\delta(k-k')\delta(m-m')}{T} \quad \ldots.\ (4.86)$$

$$\int f^*(t-mT_P)f(t-k'T_o-m'T_P)e^{-jw_Dt}dt=\frac{Te^{-jw_DmT_P}}{T}\,\delta(k')\delta(m-m') \quad \ldots.(4.87)$$

and $\int f(t-kT_o-mT_P)f(t-k'T_o-m'T_P)=\delta(k-k')\delta(m-m')\ \ldots.(4.88)$

substituting (4.86),(4.87)and (4.88)in (4.85)

$$\sum_{m=o}^{M-1} e^{-jwT_Pm}\,\delta(k')\delta(m-m')+\sum_{m=o}^{M-1}\sum_{k=o}^{N-1} g^*_{mk}\delta(k-k')\delta(m-m')$$

$$=\int g^*(t)f(t-k'T_o-m'T_P)dt \quad \ldots.\ (4.89)$$

For various values of k',m' (4.89) becomes

$$\begin{aligned}
g^*_{00}+1 &= \int g^*(t)f(t)dt\\
g^*_{10} &= \int g^*(t)f(t-T_P)dt\\
g^*_{20} &= \int g^*(t)f(t-2T_P)dt\\
g^*_{M-10} &= \int g^*(t)f(t-\overline{M-1}T_P)dt\\
g^*_{01} &= \int g^*(t)f(t-T_o)dt\\
g^*_{02} &= \int g^*(t)f(t-2T_o)dt \quad \text{and so on}
\end{aligned} \quad \ldots.\ (4.90)$$

Since $g(t)=\int_{-\infty}^{\infty} G(f)e^{-j2\pi ft}\, dt$ .... (4.91)

Substituting $G(f)$ from equation (3.58) we get

$$\int g^*(t)f(t-kT_o-mT_p)dt=\iint \frac{F_s^*(f)}{S_n^*(f)}\, e^{-j2\pi ft}\, df\; f(t-kT_o-mT_p)dt \qquad \text{.... (4.92)}$$

$$= \frac{F_s^*(f)}{S_n^*(f)}\, e^{-j(kT_o+mT_p)w}\, F(f) \qquad \text{.... (4.93)}$$

$$= \sum_{l=o}^{M-1} \frac{F^*(f+f_D)}{S_n(f)} F(f) e^{j\ell T_p(w+w_D)} \exp(-j(kT_o+mT_p)w) \qquad \text{.... (4.94)}$$

$$= \sum_{\ell=o}^{M-1} e^{j\ell T_p w_D} \int_{-\infty}^{\infty} \frac{F^*(f+f_D)F(f)e^{j(\overline{m-\ell}\, T_p+kT_o)w}}{S_n(f)}\, df \qquad \text{....(4.95)}$$

where $F(f)=\int_{-\infty}^{\infty} f(t)e^{-j2\pi ft}\, dt$ .... (4.96)

So the main task in evaluating the values of g_{mk} is to find the integrals of the form

$$\int_{-\infty}^{\infty} \frac{F^*(f+f_d)F(f)e^{-jwT_oP}}{S_n(f)}\, df \qquad \text{....(4.97)}$$

$$=\int_{-\infty}^{\infty} \frac{T\left(\frac{\sin \pi fT}{\pi fT}\right)^2 e^{-jwT_oP}}{S_{n_c}(f)+No}\, df \qquad \text{.... (4.98)}$$

where $P=0,\pm 1,\ldots MN-1$
$2N-MN-1$

where $S_{n_c}(f)$ is defined in equation.

Closed form solutions for the integrals given by (4.98) donot exist. However we can evaluate these integrals numerically. We shall consider the numerical procedure in the next chapter.

4.10 PERFORMANCE EVALUATION OF DISCRETE RESOLUTION RECEIVER.

The receiver correlation waveform corresponding to discrete resolution receiver is given by

$$g(t)=\sum_{m=o}^{M-1} f(t-mT_p)e^{-jw_D t}+\sum_{m=o}^{M-1}\sum_{k=o}^{M-1} f(t-kT_o-mT_p)g_{mk} \quad ....(4.99)$$

Let $T_p=NT_o$. The expression for the performance index Δ_D can be derived as follows:

$$\text{Now } \ell = \int_{T_i}^{T_f} r(t)g^*(t)\,dt$$

$$= \int_0^T r(t)g^*(t)dt+\int_{T_o}^{T_o+T} r(t)g^*(t)dt + ... \int_{(M-1)T_p+(N-1)T_o}^{(M-1)T_p+(N-1)T_o+T} r(t)g^*(t)\,dt$$

$$= \int_0^T r(t)\frac{[e^{jw_D t}+g_{00}]}{\sqrt{T}}\,dt+... \int_{(M-1)T_p+(N-1)T_o}^{(M-1)T_p+(N-1)T_o+T} r(t)\frac{g^*_{M-1N-1}}{\sqrt{T}}\,dt$$

.... (4.99a)

Again $e^{jw_D t}$ can be assumed to be constant over a period of T sec.

Then

$$= [\, C_0 \; C_1 \ldots C_{MN-1}] \begin{bmatrix} U_0 \\ U_1 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ U_{MN-1} \end{bmatrix} \qquad \ldots.(4.100)$$

where $C_i = \int_{mT_P+nT_o}^{mT_P+nT_o+T} r(t)dt$

$k = i \bmod M \quad \ell = j \bmod M$

$m = \frac{i-k}{M} \qquad n = \frac{j-\ell}{M}$

and $U_i = \dfrac{e^{jw_D mT_P}\delta(n) + g^*_{mn}}{\sqrt{T}}$

$$\therefore |\ell|^2 = U^{*T} C^{*T} C U$$

$$\therefore E[|\ell|^2/H_1] = U^{*T} E[C^{*T}C]\, U \triangleq U^{*T} V U \quad \text{where } V = E[C^{*T}C]$$

Now $V(i,j) = E[C_i^* C_j]$

$$= \int_{mT_P+kT_o}^{mT_P+kT_o+T} \int_{nT_P+T_o}^{nT_P+\ell T_o+T} E[r^*(t)r(u)]dtdu$$

$$= \iint [K_{n_c}(t,u) + No\delta(t-u) + |B|^2 f_s(t) f_s(u) e^{-jw_D(t-u)}]dtdu$$

Now $$\int_{mT_P+kT_o}^{mT_P+kT_o+T} \int_{nT_P+T_o}^{nT_P+T_o+T} No\delta(t-u)dtdu = NoT\delta(m-n)\delta(k-\ell)$$

$$\int_{mT_P+kT_o}^{mT_P+kT_o+T}\int_{nT_p+T_o}^{nT_P+\ell T_o+T} f_s(t)f_s(u)e^{-jw_D(t-u)}\mathcal{E}[|B|^2]\,dtdu$$

$$= T\mathcal{E}[|B|^2]\delta(\ell)\delta(k)e^{-jw_D(m-n)T_P}$$

$$\iint K_{n_c}(t,u)dtdu = K_{DR}(mT_P-nT_P)\iint R(t-u)dtdu$$

$$\therefore \qquad =K_{DR}(\overline{m-n}\,T_P)\,\frac{2}{3}\,T\delta(k-\ell)\left(\frac{M-\overline{m-n}}{M}\right)$$

$$\therefore V(i,j)=\frac{2}{3}\,\delta(k-\ell)K_{DR}(\overline{m-n}T_P)\left[\frac{M-(m-n)}{M}\right]+N_oT\delta(m-n)\delta(k-\ell)$$

$$+\,T\mathcal{E}[|B|^2]\delta(\ell)\delta(k)e^{-jw_D(m-n)T_P}$$

$$\therefore \Delta = \frac{U^{*T}\,X\,U}{U^{*T}\,Y\,U} \qquad \ldots\ldots (4.101)$$

where X,Y are MxM matrices whose elements are defined by

$$X\,(i,j) = T\delta(\ell)(k)e^{-jw_D(m-n)T_P}$$

$$Y\,(i,j) = V(i,j) - X(i,j)$$

Using equation (4.101) the performance can be evaluated. We shall consider this in the next chapter.

CHAPTER-5

PERFORMANCE EVALUATION RESULTS AND OBSERVATIONS

In this chapter the results of the numerical evaluation of the performance of the receivers considered in the previous chapters are presented. The listing of the programmes executed on DEC-10 computer are included at the end of the thesis. Along with the presentation of relevant programme outputs, assumptions made in the performance evaluation are briefly discussed.

5.1 NUMERICAL EVALUATION OF THE PERFORMANCE OF THE OPTIMUM RECEIVER.

The expression for the performance index for the optimum receiver is given in (3.62) which is reproduced here for convenience

$$\Delta_o = \overline{E}_r \int_{-\infty}^{\infty} \frac{|F_s(f-f_D)|^2}{No+S_{n_c}(f)}\, df \qquad \ldots. (5.1)$$

$$\text{where } F_s(f-f_D) = \frac{T \,\mathrm{Sin}\pi(f-f_D)T}{\pi(f-f_D)T} \sum_{i=0}^{M-1} e^{-j(w-w_D)T_P} \qquad \ldots.(5.2)$$

We evaluated (5.1) using numerical integration. The step size had been chosen so as to sample all the frequency components adequately. Since, about 90% of the area of the Sinc^2 curve is contained between $-\frac{1}{T}$, and $+1/T$ the limits of the integration in (5.1) are kept at $-\frac{N}{T}$ and $\frac{N}{T}$ and the integer N is chosen by the

following consideration. We know that the performance index corresponding to the case when clutter is absent is given by $\frac{E_r}{No}$. We computed Δ_o by numerical integration of (5.1) with a value of N selected so as to make the difference between Δ_o and E_r/No less than 2.5%.

For the computations we assumed the pulsewidth, interpulse period to be 1 μSec., $\frac{1}{540}$ seconds respectively. For 97% accuracy in the computation N was found to be two for 4 hits/scan and the number of numerical integration intervals, width of the interval were $8x10^5$, 4 respectively.

5.2 RESULTS AND OBSERVATION.

The performance indices for the conventional, mismatched, white-noise optimum, optimum receivers are computed using equations (4.35), (4.68) and (5.1) respectively.

The number of pulses in the transmitted waveform that is reflected by the target over one scan period (no of hits/scan) is taken to be four for comparing the performance of the conventional, mismatched receiver with the optimum receiver. The results of the computation are tabulated in Table 5.1.

B= 6261.8793 B+D= 9044.9368 A= 111.9096

SIGNAL	WHITE	CLUTTER	TDOPPLER	CDOPPLER	IDEAL	MATCHED	MISMATCHED
10.0	1.0	0.1	20.0	0.0	40.0	31.99664	32.04396
10.0	1.0	1.0	20.0	0.0	40.0	11.42429	11.79705
10.0	1.0	10.0	20.0	0.0	40.0	1.53769	1.69561
10.0	1.0	100.0	20.0	0.0	40.0	0.15928	0.17964
10.0	1.0	1000.0	20.0	0.0	40.0	0.01599	0.01808
10.0	1.0	10000.0	20.0	0.0	40.0	0.00160	0.00181
10.0	1.0	100000.0	20.0	0.0	40.0	0.00016	0.00018
10.0	1.0	0.1	40.0	0.0	40.0	32.70316	32.82069
10.0	1.0	1.0	40.0	0.0	40.0	12.37917	13.29301
10.0	1.0	10.0	40.0	0.0	40.0	1.71583	2.04430
10.0	1.0	100.0	40.0	0.0	40.0	0.17847	0.21767
10.0	1.0	1000.0	40.0	0.0	40.0	0.01792	0.02194
10.0	1.0	10000.0	40.0	0.0	40.0	0.00179	0.00220
10.0	1.0	100000.0	40.0	0.0	40.0	0.00018	0.00022
10.0	1.0	0.1	60.0	0.0	40.0	33.76605	33.94235
10.0	1.0	1.0	60.0	0.0	40.0	14.05374	15.61903
10.0	1.0	10.0	60.0	0.0	40.0	2.05527	2.70713
10.0	1.0	100.0	60.0	0.0	40.0	0.21549	0.29725
10.0	1.0	1000.0	60.0	0.0	40.0	0.02165	0.03004
10.0	1.0	10000.0	60.0	0.0	40.0	0.00217	0.00301
10.0	1.0	100000.0	60.0	0.0	40.0	0.00022	0.00030
10.0	1.0	0.1	80.0	0.0	40.0	35.01924	35.20229
10.0	1.0	1.0	80.0	0.0	40.0	16.51328	18.53649
10.0	1.0	10.0	80.0	0.0	40.0	2.62761	3.74963
10.0	1.0	100.0	80.0	0.0	40.0	0.27927	0.43231
10.0	1.0	1000.0	80.0	0.0	40.0	0.02810	0.04396
10.0	1.0	10000.0	80.0	0.0	40.0	0.00281	0.00440
10.0	1.0	100000.0	80.0	0.0	40.0	0.00028	0.00044
10.0	1.0	0.1	100.0	0.0	40.0	36.26097	36.39857
10.0	1.0	1.0	100.0	0.0	40.0	19.69333	21.74260
10.0	1.0	10.0	100.0	0.0	40.0	3.53674	5.17897
10.0	1.0	100.0	100.0	0.0	40.0	0.38419	0.62903
10.0	1.0	1000.0	100.0	0.0	40.0	0.03875	0.06445
10.0	1.0	10000.0	100.0	0.0	40.0	0.00388	0.00646
10.0	1.0	100000.0	100.0	0.0	40.0	0.00039	0.00065

THE RESULTS FOR OPTIMUM RECEIVER FOR THE SAME A,B,D VALUES ARE GIVEN BELOW

NO.OF SAMPLES USED FOR INTEGRATION=200000; SAMPLING INTERVAL=20HZ.

CP/SAMPLE	OPTIMUM RX.DELTA	IDEAL RX.DELTA	TARGET DOPPLER
0.1	36.94	36.97	100
1.0	26.13	36.97	100
10.0	8.96	36.97	100
100.0	2.06	36.97	100
1000.0	0.31	36.97	100
10000.0	0.04	36.97	100

TABLE 5.1

In table 5.1 the notations used have the following meaning.

SYMBOL	MEANING
SIGNAL	TARGET SIGNAL power/Sample
WHITE	Spectral density of white noise
CLUTTER	Clutter power/sample
TDOPPLER	Target doppler frequency
CDOPPLER	Doppler freq. due to clutter
IDEAL	Δ_i-THE PERFORMANCE INDEX FOR OPTIMUM RECEIVER IN THE Absence of clutter.
MATCHED	for conventional receiver
MISMATCHED	for mismatched receiver.

From table 5.1 we observe the following :

1) Δ for the white-noise optimum (denoted as ideal) receiver should be 40 for the given signal and noise parameters. But Δ', the value of Δ computed using numerical procedure is 38.97. Hence the numerical procedure has about 2.5% inaccuracy.

2) For the same signal and white noise powers. The performance of optimum receiver is superior to be other two receivers for all clutter powers and for all target doppler shifts, as it should be.

3) The superiority of the optimum Rx increases with increasing clutter power. This may be verified by computing $\frac{\Delta_o - \Delta_c}{\Delta_c}$, $\frac{\Delta_o - \Delta_m}{\Delta_m}$. The result of such an analysis is given in Table 5.2 for some typical values from table 5.1.

TABLE 5.2

COMPARISON OF SUBOPTIMUM RECEIVERS PERFORMANCE INDICES WITH THE OPTIMUM RX.

TARGET DOPPLER = 100 Hz

Target Signal Power= 10

White noise No = 1

Spectral density

B = 6261. 8793

B+D= 9044. 9368

A = 111. 9096

CLUTTER POWER	[redacted]	[redacted]	mismatched	% improvement in in optimum Rx compared to	
				Conventional Rx	Mismatched Rx.
0.1	36.93	36.26	36.39	1.8	1.4
1.0	26.17	19.69	21.74	32.9	20.3
10.0	8.95	3.53	5.17	153.5	73.1
100.0	2.06	0.38	0.62	442.1	232.2
1000.0	0.34	0.03	0.06	1033.0	466.0
10000.0	0.04	0.003	0.006	1233.3	566.0

4) At large dopplers the difference in performance of the above receivers is small.

We obtained the receiver operating characteristics viz the plot of probability of detection Vs probability of False alarm and are given in Fig.5.1-5.6 for some representative values.

The computational burden involved in evaluating the performance of the optimum receiver for the case of 16 hits per scan is excessive. Therefore, for the case of 16 hits per scan. We limit our comparison of performance of the above structured receivers' with that of the receiver that would be optimum in the absence of clutter. The results of the computation are tabulated in table 5.3. From the table 5.3 we observe the following :

1) When the relative doppler between the clutter and the target increases the performance improves.
2) We observe that increasing target power improves the performance.
3) When the noise is predominantly white both the receivers have the same performance. Otherwise mismatched Rx is superior to the conventional Rx.
4) As the clutter power increases the superiority of the mismatched Rx over the conventional Rx improves.

ROC's for some typical values of Δ are shown in Fig. 5.7-5.10.

We have not attempted to compute the performance of the discrete resolution receiver due to the relatively large computational time requirements.

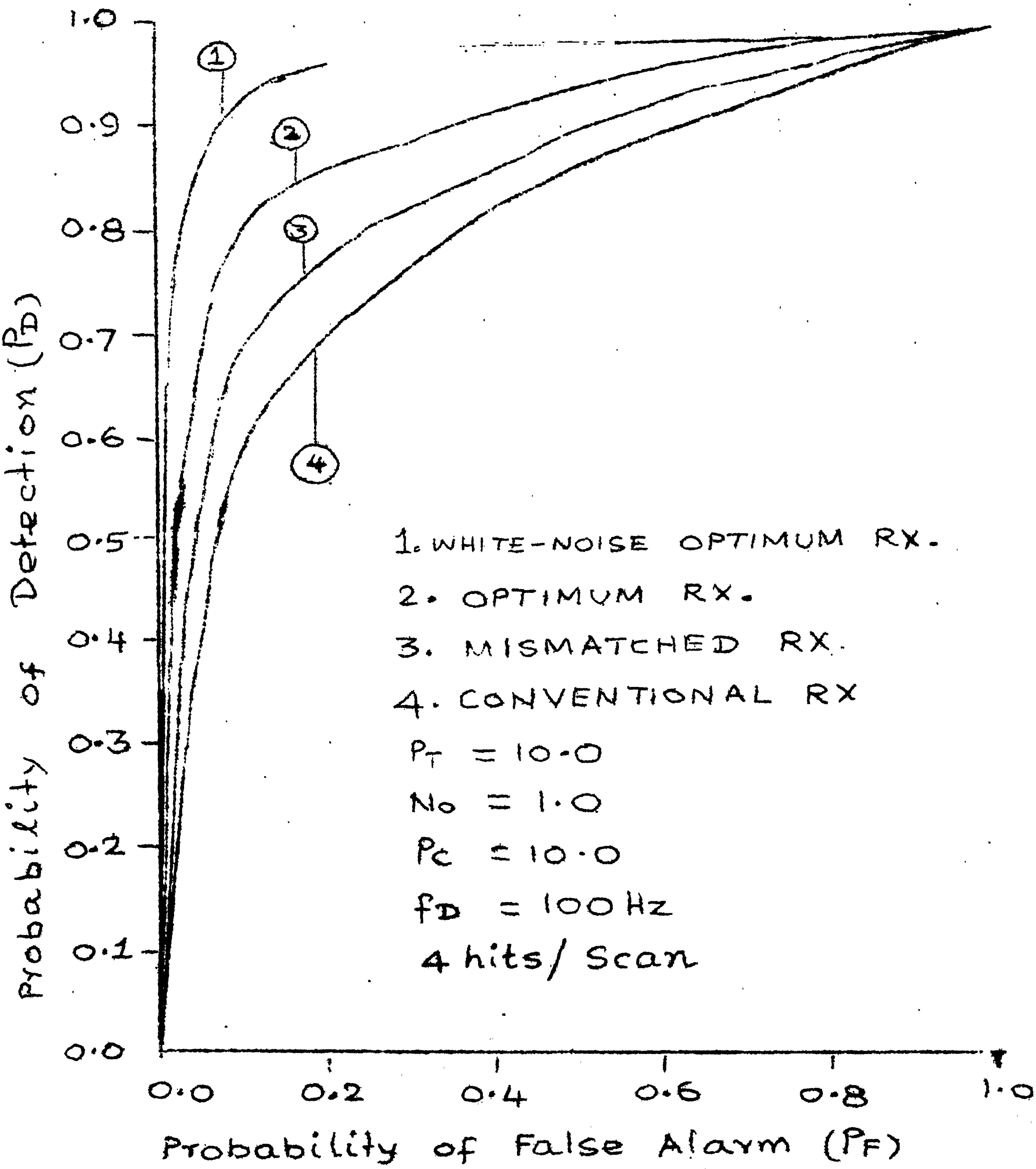


FIG. 5.1 . RECEIVER OPERATING CHARACTERISTICS FOR OPTIMUM AND STRUCTURED - OPTIMUM RECEIVERS

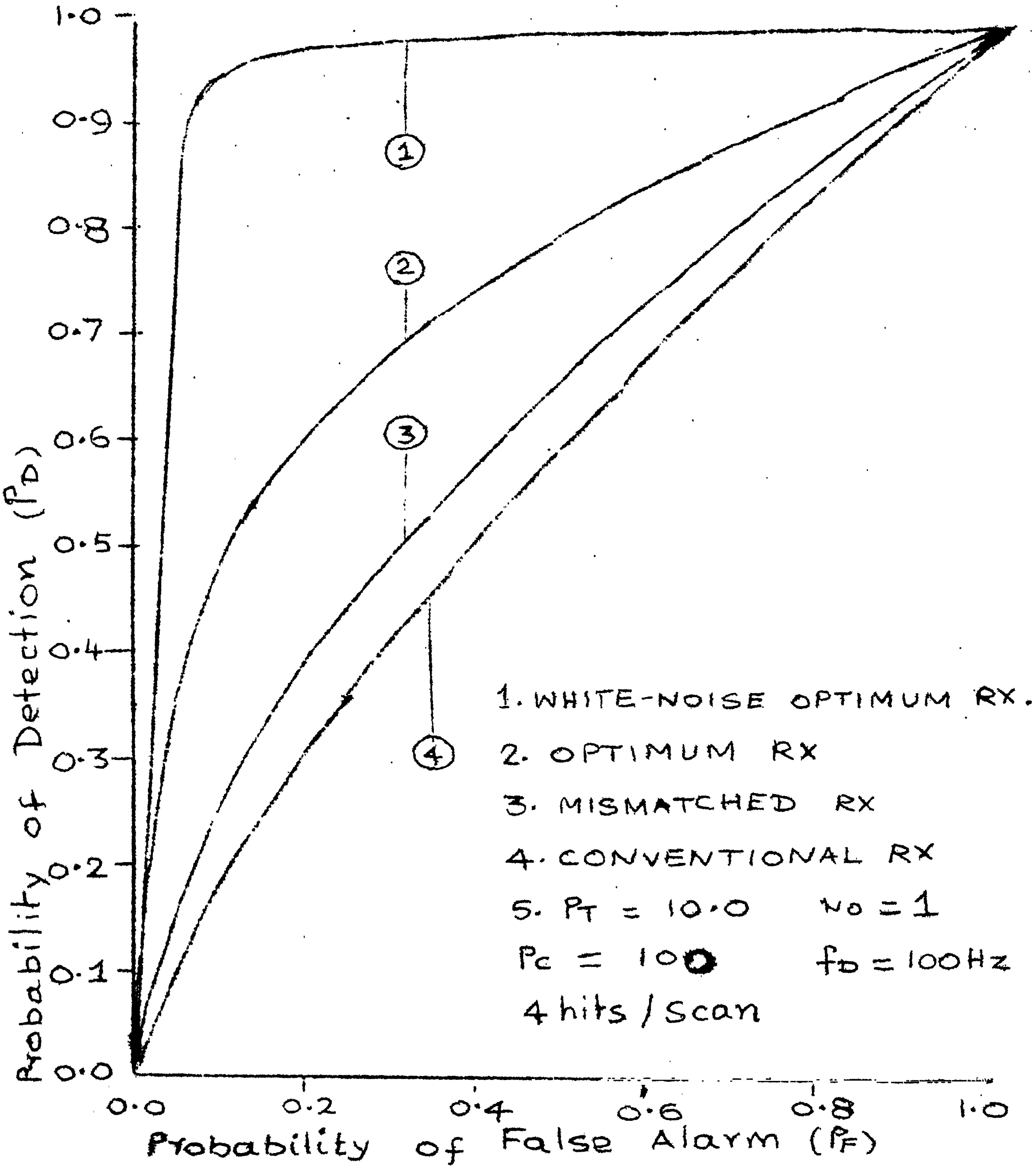


FIG. 5.2 RECEIVER OPERATING CHARACTERICS OF OPTIMUM AND STRUCTURED OPTIMUM RECEIVERS.

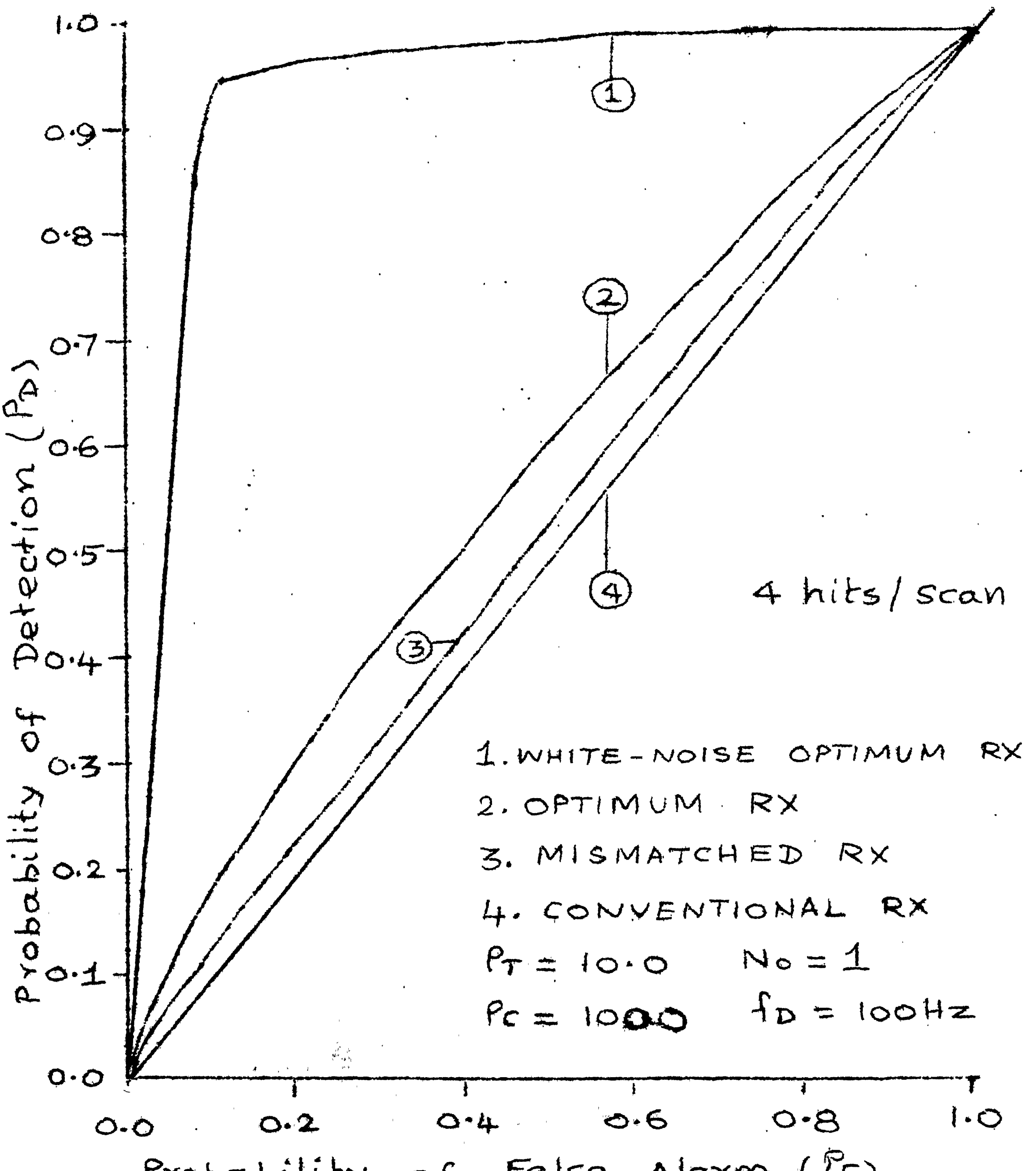


FIG. 5.3 RECEIVER OPERATING CHARACTERISTICS OF OPTIMUM AND SUBOPTIMUM RECEIVERS.

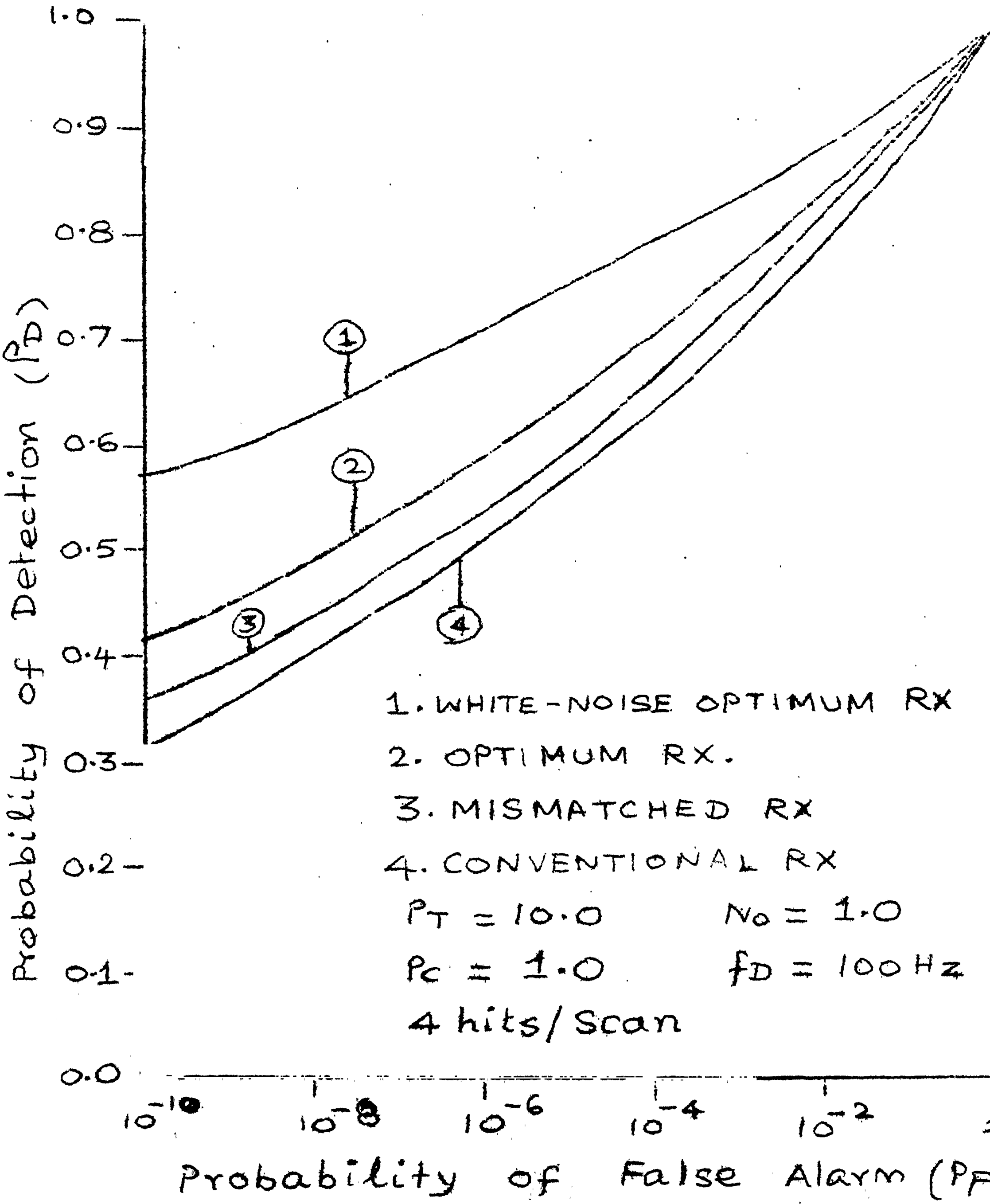


FIG. 5.4 RECEIVER OPERATING CHARACTERISTICS OF OPTIMUM AND STRUCTURED-OPTIMUM RECEIVERS

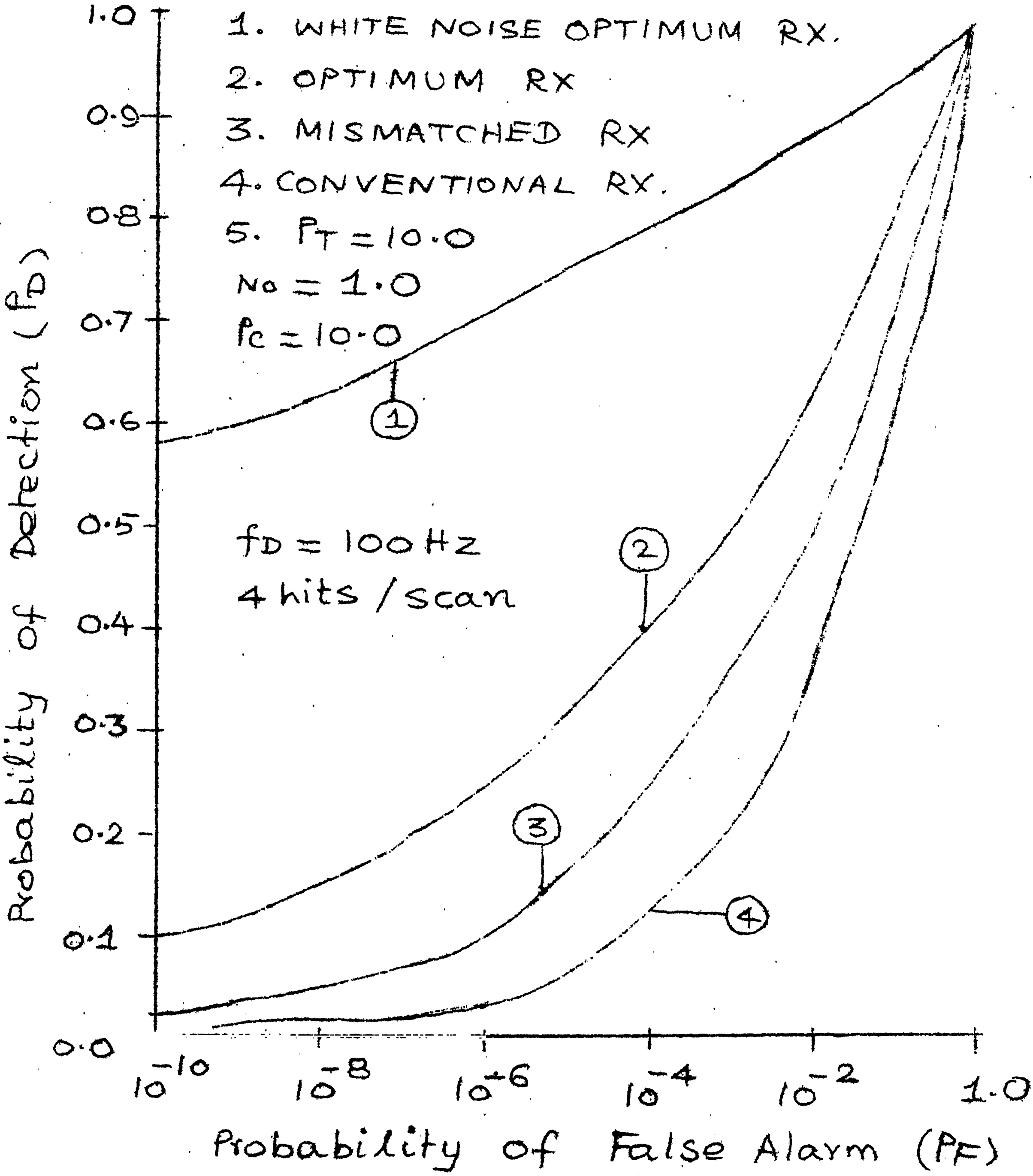


FIG. 5.5 RECEIVER OPERATING CHARACTERISTICS OF OPTIMUM AND STRUCTURED - OPTIMUM RECEIVERS

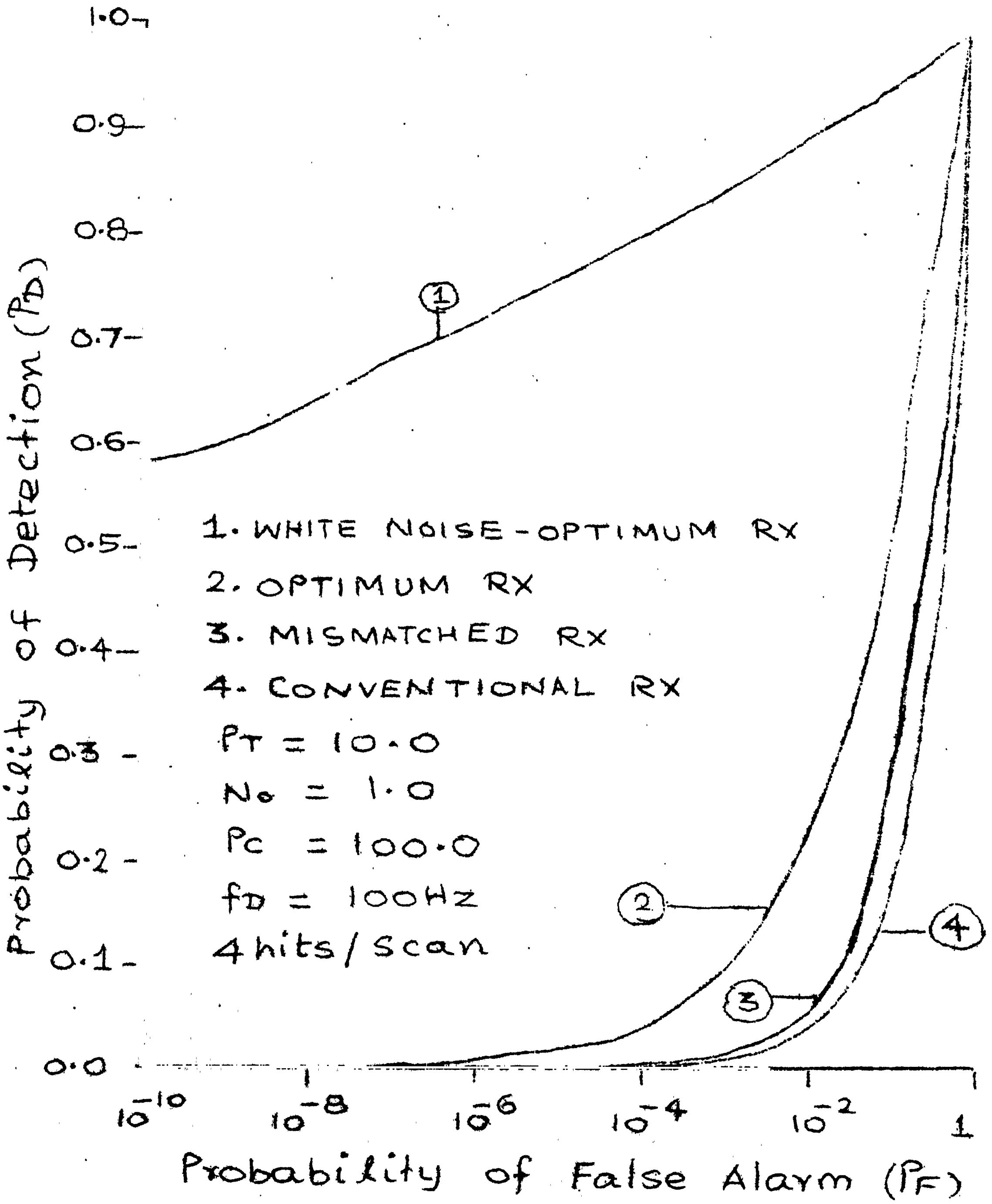


FIG. 5.6 RECEIVER OPERATING CHARACTERISTICS OF OPTIMUM AND STRUCTURED-OPTIMUM RECEIVERS

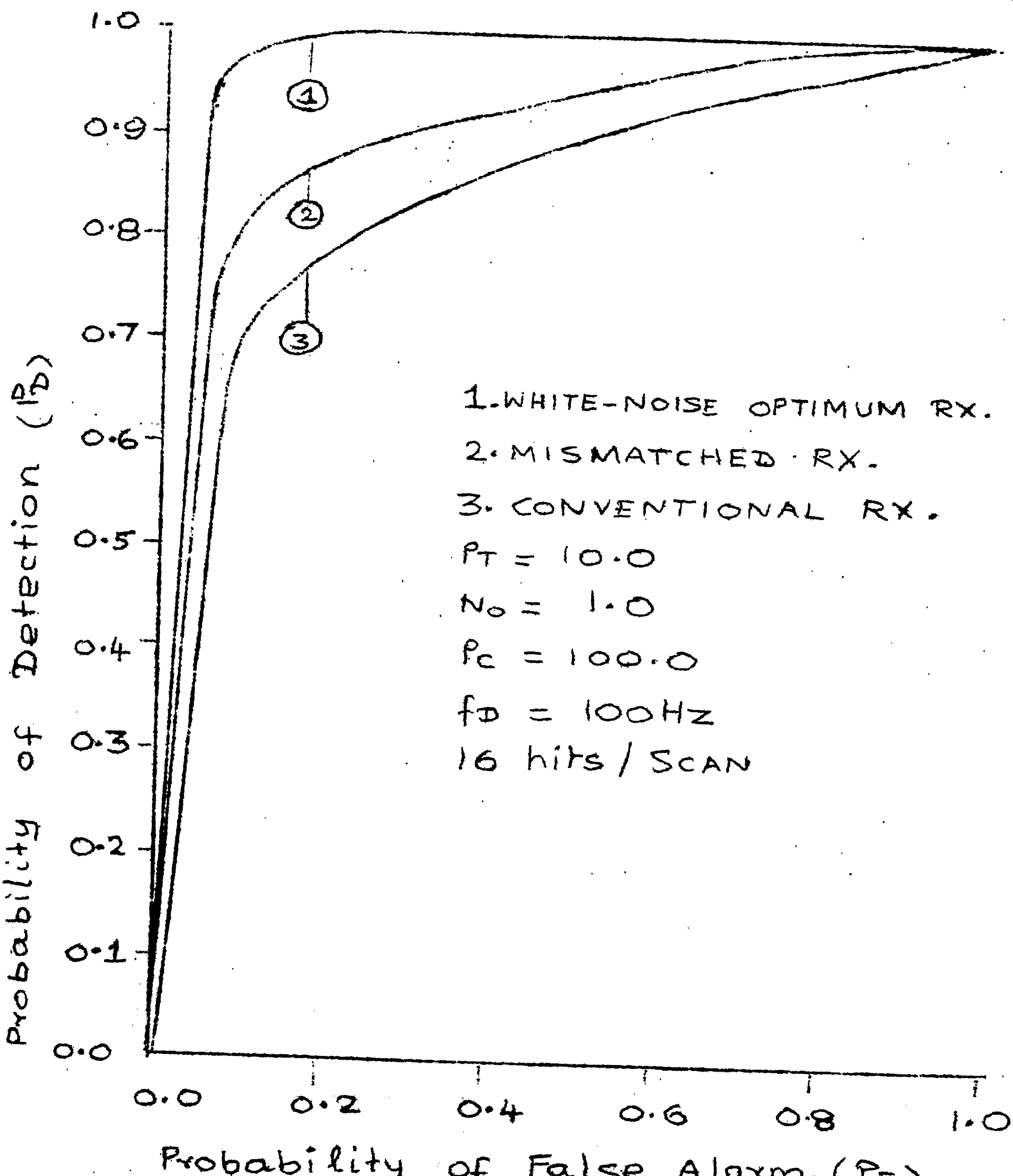


FIG. 5.7. RECEIVER OPERATING CHARACTERISTICS FOR WHITE-NOISE OPTIMUM AND STRUCTURED OPTIMUM RECEIVERS.

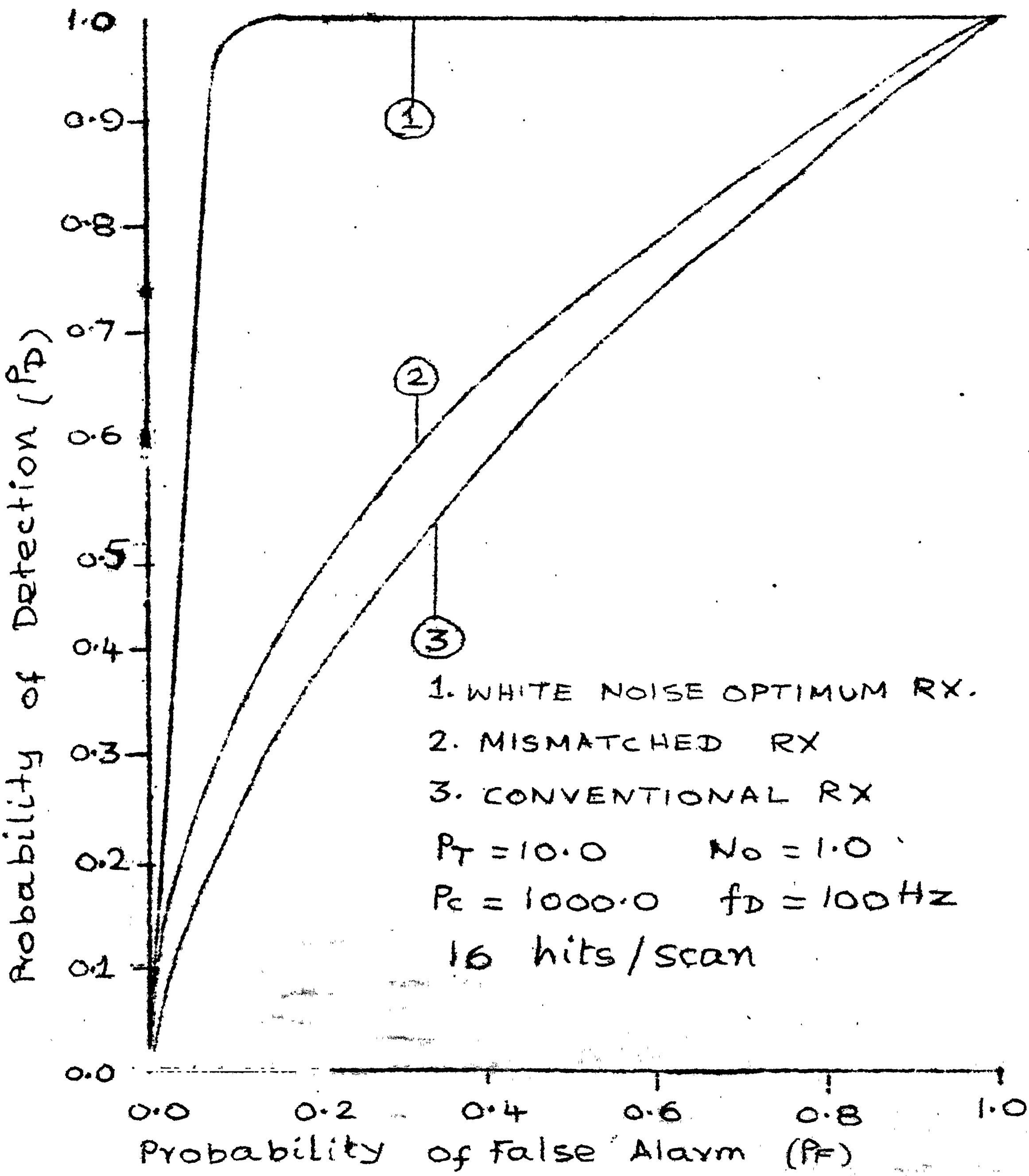
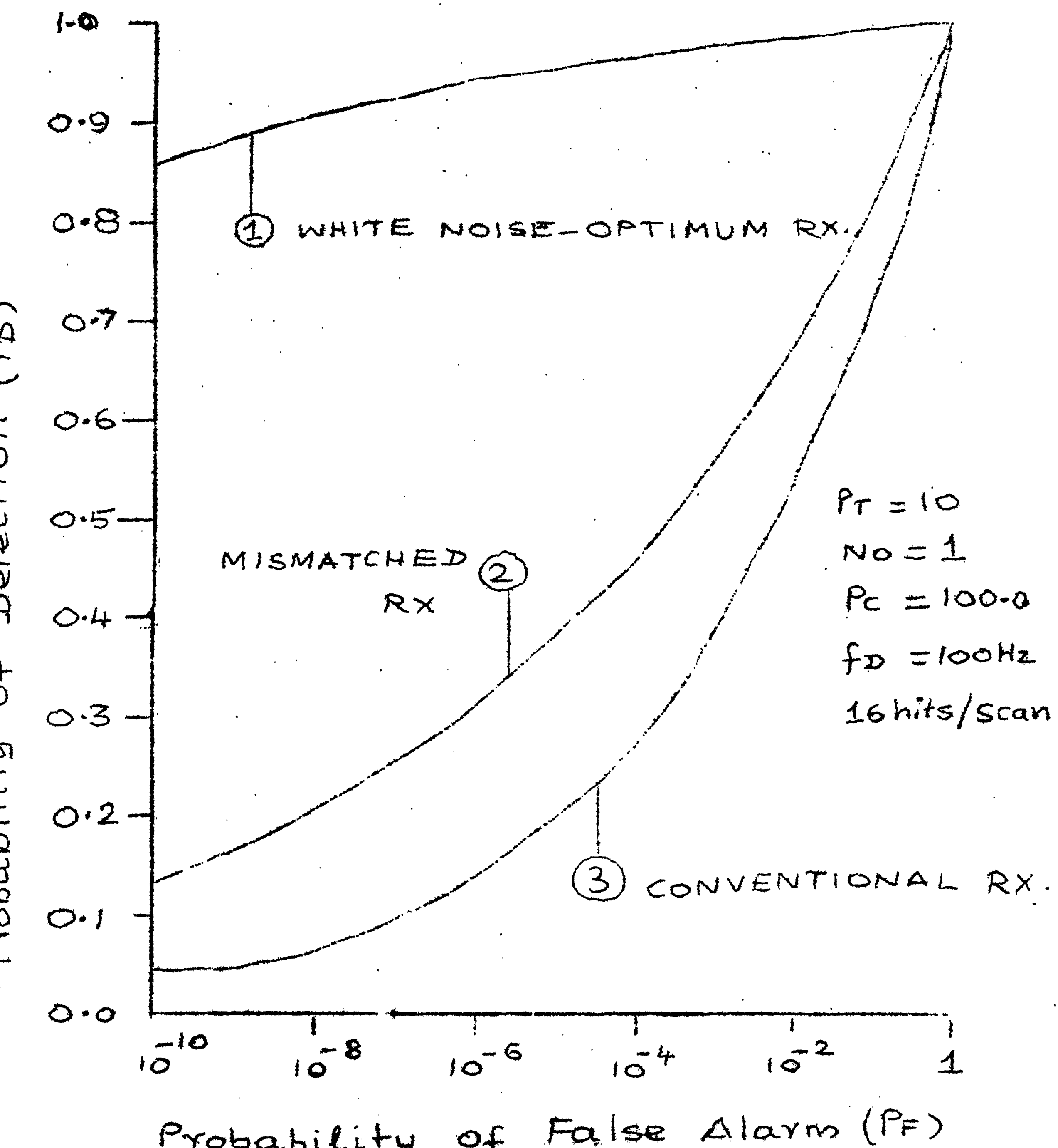


FIG 5. RECEIVER OPERATING CHARACTERISTICS OF WHITE-NOISE OPTIMUM AND STRUCTURED-OPTIMUM RECEIVERS



RECEIVER OPERATING CHARACTERICS OF WHITE NOISE-OPTMUM AND STRUCTURED OPTIMUM RECEIVERS. FIG. 5.9

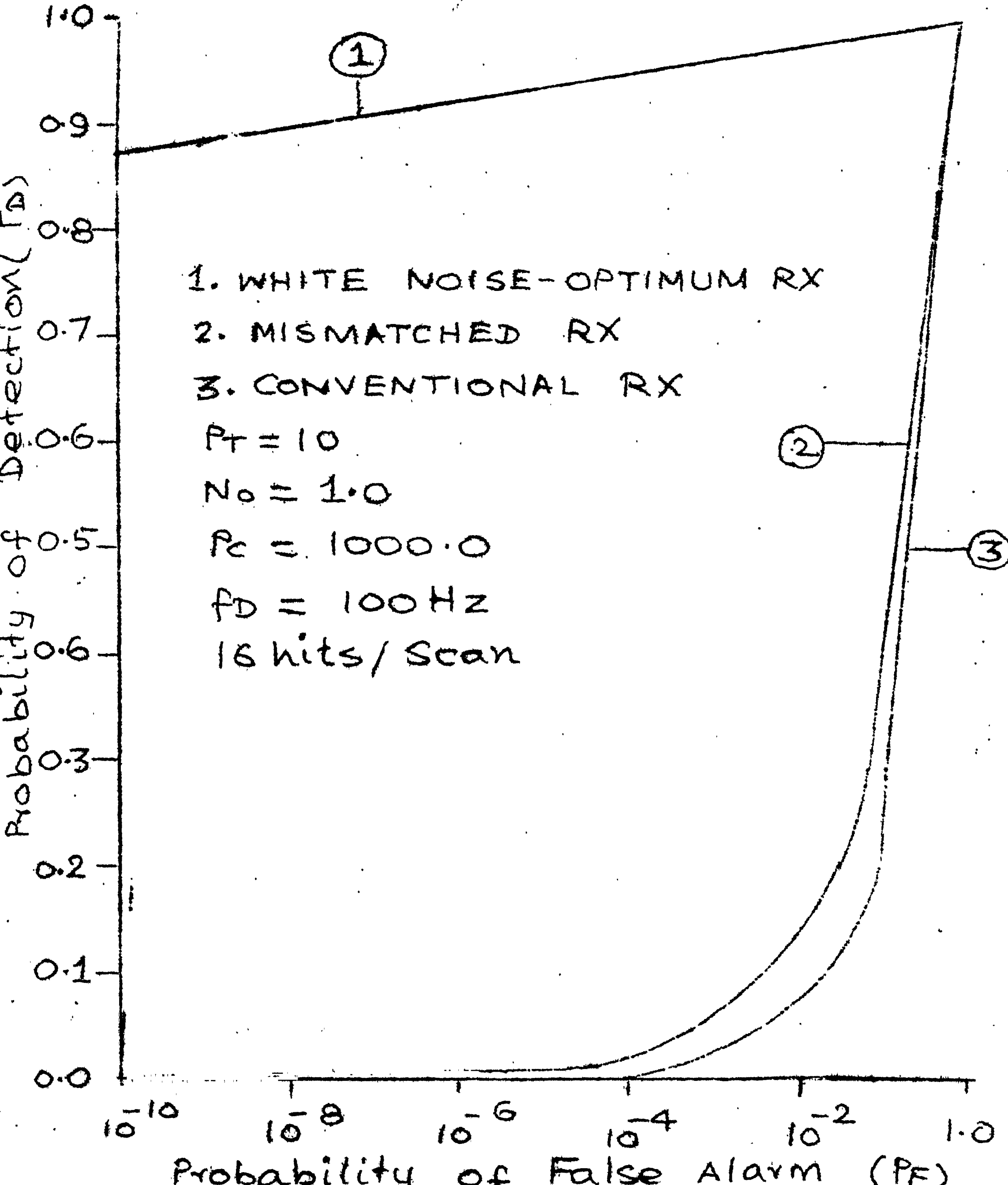


FIG. 5·10. RECEIVER OPERATING CHARACTERISTICS OF WHITE NOISE-OPTIMUM AND STRUCTURED-OPTIMUM RECEIVERS

CHAPTER-6

CONCLUSION AND SUGGESTIONS FOR FURTHER WORK

6.1 CONCLUSION.

In this thesis we studied some receivers for a pulsed radar to detect a Swerling 1 target in the presence of a random dipole modelled clutter. The clutter model incorporated is representative of chaff and vegetation clutter characterized by a spectrum with a secular component arising from the rotation of the scatterers. For the pulsewidths and pulse repetition frequencies normally used in a pulsed radar, the aircraft can be modelled as a slowly fluctuating point target and we assumed a Swerling 1 target.

For the above target detection problem we attempted to solve the associated integral equation. Although we succeeded in finding a solution involving complex error functions, we found it too unwieldy to be of any practical interest. We then derived an approximate solution under the realistic assumption that the clutter correlation function can be treated as a constant over intervals of one pulsewidth of the transmitted pulse train. However this solution only gives us the Fourier transform of the time signal whose cross correlation with the received signal the optimum receiver would evaluate before thresholding and making a decision about the presence or absence of the target. As an analytic expression for the

inverse Fourier transform which would have in an explicit form, the dependance of the required time signal on the clutter parameters, escaped our efforts we did not consider any further the implementation aspects of the optimum receiver. However, to allow for a comparative evaluation of the performance of other receivers with that of the optimum receiver, we obtained an expression for the optimum receiver performance.

In view of the above observations about the optimum receiver we next considered three relatively easily implementabl structured-optimum receivers. They are the conventional receiver, the mismatched receiver and the discrete resolution receiver. An adaptive version of the mismatched receiver was proposed for changing clutter environment. In this connection the parameters ($\sigma_d, \sigma_r, V_{or}, V_{od}$) were quantized and for ensuring the degradation in performance due to quantization to be below 0.1% it was found that (8,2,2,8) levels were sufficient. We concluded that the parameters can be estimated by computing the clutter spectrum and then using the nearest neighbourhood method.

The performances of the mismatched and conventional receivers were evaluated for various combinations of signal, white noise clutter power and for different clutter and target dopplers for 16 hits/scan. Their performances were

compared with the white-noise optimum receiver in the absence of clutter. The mismatched receiver performs better than the conventional receiver at large clutter power. When the noise is predominantly white their performances are the same. Their performances were compared with the optimum receiver in the presence of clutter at 4 hits/scan. Their performance is 'satisfactory' at large target dopplers. The performance of the discrete resolution receiver could not be evaluated because of excessive computational requirements.

6.2 SUGGESTION FOR FURTHER WORK.

As an extension of the work done in this thesis a comparative study of the receivers proposed in this thesis and that considered by Haykins et al [19] may be undertaken. Haykins et al [19] uses the random dipole clutter model for designing an MTI Radar receiver with constant False Alarm rate. If the nth order FIR filter used in this receiver is designed to maximize the probability of detection performance then such a comparison is feasible.

Further, if the clutter parameters are quantized then the solution to the integral equation associated with the optimum receiver may be numerically evaluated to store the corresponding optimum filter coefficients for each possible

combination of the quantized parameter values. Then in a changing clutter environment as soon as the clutter parameters are estimated the optimum filter coefficients may be used to process the received signal.

In stead of using the random dipole model for clutter directly as we did in this thesis and as suggested above one may fit an Nth order ARMA model to the clutter. The value of N for representing the clutter covariance within reasonable accuracy may be found to implore the implementability and adaptability of the associated whitening filter.

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```
PROGRAM R66.FOR
THIS  PROGRAM COMPUTES THE % DEGRADATION
IN PERFORMANCE DUE TO QUANTIZATION OF THE
PARAMETERS A,B,C,D
COMPLEX RES,RE,ST,UST,G,G1
COMMON AK,X,Y,Z,EN0,AN,TPWD1,ST,UST
DIMENSION  X(20),Y(20),Z(20),ST(20),TO(20)
DIMENSION RE(20),ST(20),UST(20),AK(16,16)
DIMENSION G(20),G1(20)
NOP=16
PRINT 101
DO 350 NN=1,3
DO 350 N5=1,3,2
P1=N5-2
P2=0
P3=NN-3
N2=4.0-P3
N3=10.0-P3
Maximum values of A,B,D, are given by AA,BB,BBD
AA=4.0*3.14*0.033*540.0
BB=8.0*(3.14*540.0*0.033)**2
BBD=2.0*(3.14*540.0*0.044)**2
B1=BB/10.0
D1=BBD/10.0
A1=AA/12.0
SNS=0.01
EN0=1.0
DO 350 J=7,7
SN=SNS*(10.0**J)
DO 350 N1=1,3
DO 350 LL=2,8,3
AL=LL
B=B1*AL
EN1=1.0
EN0=white noise power/sample
DO 350 JJ=N1,N11
DO 350 NS=N2,N3,6
AN=NS
AJJ=JJ

D=D1*AJJ
BD=B+D
A=AN*A1
TP=interpulse period
T=pulsewidth in seconds
TP=1.0/540.00
FD=100.0
TPWD1=TP*6.28*FD
BTP1=B*TP*TP
DTP1=BD*TP*TP
ATP=A*TP
The value of exp(-BTP.TP.K.K), exp(-(B+D).TP.TP.K.K)
precomputed for various values of K.
TYPE *,N11
AM=NOP
DO 10 I=1,NOP
AI=I-1
BSQT=-1.0*(AI*AI*BTP1)
DSQI=-1.0*AI*DTP1*AI
X(I)=EXP(BSQT)
Y(I)=(EXP(DSQI)*COS(AI*ATP)+0.5+X(I))*(AM-AI)/AM

SIG=10.0
CDOP=0.0
```

```
C I        (G,CD  ,S )

DO       =        
G (  )= (  )
BO=
DO=
AO=

B=B1*(  +P1)
D=D1*(   +P2)
BD=B+D
A=A1*(  +P3)
BTP1=B*TP*TP
DTP1=D*TP*TP
ATP=TP*A
DO 14 I=1,NOP
AI=I-1
BSOT=-1.0*(AI*AI*BTP1)
DSOI=-1.0*AI*DTP1*AI
X(I)=EXP(BSOT)
X(I)=(EXP(DSOI)*COS(AI*ATP)*0.5+X(I))*(AM-AI)/AM
CALL KNMAT(G,CDUP,SM)
GO TO 5500
PRINT 103,B,BD,A
CALL RESUL2(BO,B,DO,D,AO,A,STG,SM,G,G1,CDUP)
PRINT 102
FORMAT(6X,'OLDB        NEWB          OLDD      NEWD        OLDA
    NEWA    MATCHED   MISMATCHED1     MISMATCHED2    %ERROR')
FORMAT(1X)
FORMAT(5X,'B=',F10.4,5X,'B+D=',F10.4,5X,'A=',F10.4)
STOP
END


SUBROUTINE RESUL2(BO,B,DO,D,AO,A,STG,SM,G,G1,CDUP)
COMPLEX RES,RE,ST,UST,G,GTEMP,CST,CUST,G1,GTEM1

COMMON AK,X,Y,Z,TWO,AM,TPWD1,ST,UST
DIMENSION  X(20),Y(20),Z(20),ST(20),TO(20)
DIMENSION RE(20),ST(20),UST(20),AK(16,16),CST(20),CUST(20)
DIMENSION G(20),GTEMP(20),G1(20),GTEM1(20)
NOP=AM
DO 1020 I=1,NOP
CUST(I)=UST(I)
CST(I)=ST(I)
RES=(0.0,0.0)
DO 100 K=1,NOP
RES=RES+AK(I,K)*UST(K)
RE(I)=RES
FDS=TPWD1/(6.28/540.0)
RES=(0.0,0.0)
DO   1021  I=1,NOP
RES=ST(I)*RE(I)+RES
RES1=RES
DO 1220  I=1,NOP
RES=(0.0,0.0)
TYPE *,G1(I),G(I)
DO     K=1,NOP
RES=RES+AK(I,K)*G(K)
RE(I)=RES
RES=(0.0,0.0)
DO    1221  I=1,NOP
GTEMP(I)=CONJG(G(I))
RES=GTEMP(I)*RE(I)+RES
RES2=RES
DO  1120  I=1,NOP
RES=(0.0,0.0)
DO  1210  K=1,NOP
RES=RES+AK(I,K)*G1(K)
RE(I)=RES
```

```
RES3=RES
TYPE *,RES1,RES2,RES3
TPWOP=TPW01
TPW01=COOP*6.28/540.0
CALL DOPEFF
TPW01=TPW00
RE(1)=(0.0,0.0)
RE(3)=([illegible])
RE(2)=RE(1)
DO 40 I=1,NOP
RE(1)=RE(1)+UST(I)
RE(3)=RE(3)+G1(I)/CUST(I)
RE(2)=RE(2)+G(I)/CUST(I)
ST(1)=ST(1)*RE(1)
GTEMP(1)=GTEMP(1)*RE(2)/CST(1)
GTEM1(1)=GTEM1(1)*RE(3)/CST(1)
DO 41 I=2,NOP
ST(1)=ST(1)+ST(I)*RE(1)
GTEM1(1)=GTEM1(1)+GTEM1(I)*RE(3)/CST(1)
GTEMP(1)=GTEMP(1)+GTEMP(I)*RE(2)/CST(1)
STG1=ST[illegible]*ST(1)
RES([illegible]=STG1/([illegible]*[illegible])
RES1=ST[illegible]/[illegible]S1
STG2=ST[illegible]*[illegible](1)
RES2=ST[illegible]/[illegible]S2
STG3=ST[illegible]*[illegible](1)
RES3=[illegible]/[illegible]S3
PER=1[illegible]*(RES2-RES3)/RES2
PR[illegible],NU,A,RES1,RES2,RES3,PER
F[illegible](1X,[illegible].1,F10.1,1F10.1,2F8.1,F11.5,3X,F10.5,F15.5,F10.1)
RETURN
END
```

```
PROGRAM 077.FOR
PROGRAM FOR EVALUATION OF PERFORMANCE OF
MULTIPULSE OPTIMUM,CONVENTIONAL,MISMATCHED
PRECISION
COMPLEX QRS,RE,ST,UST,G,C1
COMMON AK,X,Y,Z,FAC,AM,TPWD1,ST,UST
DIMENSION X(20),Y(20),Z(20),ST(20),TO(20)
DIMENSION RE(20),ST(20),UST(20),AK(16,16)
DIMENSION G(20)
NOP=4
Maximum values of A,B,D, are given by AA,BB,BBD
AA=4.0*3.14*0.033*540.0
BB=8.0*(3.14*540.0*0.033)**2
BBD=2.0*(3.14*540.0*0.044)**2
DO 350 LL=1,1
AL=LL
B=BB*AL/4.0
EN1=1.0
EN0=white noise power/sample
DO 350 JJ=1,1
DO 350 NN=1,1
AN=NN
AJJ=JJ
BD=(BBD*AJJ/4.0)+B
A=AA*AN/2.0
PRINT 102
PRINT 103,B,BD,A
PRINT 101
TP=interpulse period
T=pulse width in seconds
...
The value of exp(-BTP.TP.K.K), exp(-(B+D).TP.TP.K.K)
precomputed for various values of K.
AM=NOP
DO 10 I=1,NOP
AI=I-1
BSQT=AI*AI
TYPE *,BTP1
BSQT=-1.0*(  BSQT*BTP1)
TYPE *,BSQT
DSQI=-1.0*AI*DTP1*AI
X(I)=EXP(BSQT)
X(I)=(EXP(DSQI)*COS(AI*ATP)*0.5+X(I))*(AM-AI)/AM

DO 350 ...=1,6
SNN=...
SIG=10.0
EN0=1.0
CDOP=0.0
SN=1000.0
AM=NOP

TPWD1=TPWD
GO TO 55
DO 332 I=1,5
CDOP=CDOP+10.0
CALL AMBAT(G,CDOP,SN)
CALL ...(SIG,SN,TP,SI,0,...)
...
TPWD=TPWD/10.0
```

```
DO 335 I=1,5
AI=I
TPWD1=TPWD*AI
CALL KUMAT(G,CDOP,SN)
CALL RESULT(SIG,SN,TO,SI,G,CDOP)
PRINT 102
TPWD1=TPWD*10.0

SIGI=0.1
SN=1.0
ENO=EN1
TPWD1=TPWD
DO 330 I=1,2
SIG=SIGI*(10.0**I)
CALL KUMAT(G,CDOP,SN)
CALL RESULT(SIG,SN,TO,SI,G,CDOP)
PRINT 102
SIG=10.0
DO 33 I=1,5
ENO=EN1*(10.0**I)
CALL KUMAT(G,CDOP,SN)
CALL RESULT(SIG,SN,TO,SI,G,CDOP)
PRINT 102
SNS=0.01
ENO=1.0
TPWD1=TPWD
CALL DOPEFF
DO 350 J=1,7
SI=SNS*(10.0**J)
CALL KUMAT(G,CDOP,SN)
CALL RESULT(SIG,SN,TO,SI,G,CDOP)
PRINT 102
FORMAT(' SIGNAL        WITH     CLUTTER   TDOPPLER    CDOPPLER
   1          MATCHED            MISMATCHED')
FORMAT(1X)
FORMAT(10X,'S=',F10.4,5X,'N+D=',F10.4,5X,'A=',F10.4)
STOP
END



SUBROUTINE RESULT(SIG,SN,TO,SI,G,CDOP)
COMPLEX RES,RE,ST,UST,G,GTEMP,CST,CUST

COMMON AM,X,Y,Z,ENO,EN,TPWD1,ST,UST
DIMENSION X(20),Y(20),Z(20),SI(20),TO(20)
DIMENSION RE(20),ST(20),UST(20),AK(16,16),CST(20),CUST(20)
DIMENSION G(20),GTEMP(20)
NOP=AM
DO 1020 I=1,NOP
CUST(I)=UST(I)
CST(I)=ST(I)
RES=(0.0,0.0)
DO 100 K=1,NOP
RES=RES+AK(I,K)*UST(K)
RE(I)=RES
FDS=TPWD1/(6.28/540.0)
RES=(0.0,0.0)
DO 1021 I=1,NOP
RES=ST(I)*RE(I)+RES
RES1=RES
DO 120 I=1,NOP
RES=(0.0,0.0)
DO 120 K=1,NOP
RES=RES+AK(I,K)*G(K)
RE(I)=RES
RES=(0.0,0.0)
DO 1022 I=1,NOP
GTEMP(I)=CONJG(G(I))
RES=GTEMP(I)*RE(I)+RES
RES2=RES
TPWD1=TPWD1
TPWD1=TPWD*6.28/540.0
CALL DOPEFF
TPWD1=TPWD
```

```
DO [illegible] I=1,[illegible]
[illegible](1)=[illegible](1)+[illegible]ST(I)
RE(2)=[illegible]+G(I)/CUST(I)
S[illegible](1)=[illegible](1)*R[illegible](1)
G[illegible]P(1)=[illegible]P(1)*RE(2)/CST(1)
DO [illegible] I=[illegible]
S[illegible](I)=ST(I)+S[illegible](I)*RE(1)
GTEMP(I)=[illegible](1)+GTEMP(I)*RE(2)/CST(I)
SIG1=[illegible]*S[illegible](1)
RES1=[illegible]1/([illegible]*[illegible])
RES1=[illegible]/[illegible]
SIG2=[illegible]*[illegible]P(1)
RES2=[illegible]/[illegible]S2
PRINT [illegible],SIG,[illegible],EPS,CDUP,RES0,RES1,RES2
FORMAT([illegible]X,2F11.1,2F11.1,2F11.1,F11.1,F11.5,3X,1F1[illegible].5)
RETURN
END
```

```
      PROGRAM R776.FOR
      SUBROUTINE KMAT(G,CDOP,SO)
      COMPLEX ST,UST,G,RES
      COMMON AK,P,Q,Z,ENO,AM,TPWD1,ST,UST
      REAL P,Q,X,Y
      DIMENSION P(20),Q(20)
      DIMENSION  X(20),Y(20),Z(20),G(20)
      DIMENSION RE(20),ST(20),UST(20),AK(16,16)
      NOP=AM
      DO 11 I=1,NOP
      X(I)=P(I)*SO
      ACT=0.5
      CALL DOPEFF
      GO TO 900
      CALL INVS(AK,NOP)
      DO 800 I=1,NOP
      RES=(0.0,0.0)
      DO 700 J=1,NOP
      RES=RES+AK(I,J)*UST(J)
      G(I)=RES
      ACT=0.66
      DO 1000 I=1,NOP
      DO 1000 J=1,NOP
      IJ=I-J
      IF(IJ.GE.0) GO TO 1001
      IJ=-IJ
1     AIJ=IJ
      IJ=IJ+1
      AK(I,J)=ACT*X(IJ)
      IF(IJ.EQ.1) AK(I,J)=AK(I,J)+ENO
0     CONTINUE
      IF (ACT.EQ.0.5) GO TO 600
      TPWD0=TPWD1
      TPWD1=(CDOP*6.28/540.0)+TPWD1
      CALL DOPEFF
      TPWD1=TPWD0
      RETURN
      END


      SUBROUTINE DOPEFF
      COMPLEX ST,UST
      COMMON AK,X,Y,Z,ENO,AM,TPWD1,ST,UST
      DIMENSION  X(20),Y(20),Z(20)
      DIMENSION RE(20),ST(20),UST(20),AK(16,16)
      NOP=AM
      UST(1)=(1.0,0.0)
      ST(1)=UST(1)
      DO 1010 I=2,NOP
      AI=I-1
      COO=TPWD1*AI
      CO=COS(COO)
      C1=SIN(COO)
      UST(I)=CMPLX(CO,C1)
0     ST(I)=CONJG(UST(I))
      RETURN
      END

      SUBROUTINE INVS(AA,NDV)
      DIMENSION AA(NDV,NDV)
      DO 20 K=1,NDV
      PVT=AA(K,K)
      AA(K,K)=-1./AA(K,K)
      DO 5 I=1,NDV
      IF(I-K)3,5,3
      AA(I,K)=-AA(I,K)*AA(K,K)
      CONTINUE
      DO 4 I=1,NDV
      DO 4 J=1,NDV
      IF((I-K)*(J-K))9,4,9
      AA(I,J)=AA(I,J)-AA(I,K)*AA(K,J)
      CONTINUE
      DO 20 J=1,NDV
      IF(J-K)18,20,18
      AA(K,J)=-AA(K,J)*AA(K,K)
      CONTINUE
      DO 25 I=1,NDV
      DO 25 J=1,NDV
      AA(I,J)=-AA(I,J)
```

```
PROGRAM DP22.FOR
PROGRAM FOR EVALUATION OF OPTIMUM PERFORMANCE
THIS TAKES ABOUT 2.42 MINUTES FOR M=4 FOR
CALCULATING DELTA ONCE.THIS PROGRAM IS VALID
FOR ANY TARGET DOPPLER FREQ.SHIFT
COMPLEX RES,RC,ST,UST
REAL X,Y,Z,P,Q
DIMENSION  X(20),Y(20),Z(20),P(20),Q(20)
DIMENSION RE(16),ST(20),UST(20)
T=10.0**-6
MOD=1
A=[illegible]
T[illegible]=20000.0
TIP=(10.0**6)/540.0
FLIF=150.0
NO=200
FN=1.0
S[illegible]=0.01
SIG=10.0
FD=80.0
H=FINIT
maximum values of A,B,D, are given y AA,BB,BBD
AA=4.0*3.14*0.033*540.0
BB=8.0*(3.14*540.0*0.033)**2
BBD=2.0*(3.14*540.0*0.044)**2
DO 350 LL=1,1
AL =LL
B=BB*AL/4.0
[illegible]
[illegible]
[illegible]
[illegible]
[illegible]
B[illegible]=([illegible])**3
[illegible]
[illegible]
[illegible]
[illegible]
FORMAT([illegible],'A=',F11.3,[illegible],'B=',F11.3,[illegible],'B+D=',F11.3)
FORMAT([illegible]                    IDEAL  DOPPLER')
FORMAT([illegible],F10.1,[illegible],F5.2,1X,F5.1)

[illegible]
[illegible]
[illegible]
[illegible]
[illegible]
[illegible]
[illegible]
RSQ[illegible]=-1.[illegible]
RSQ1=-1.[illegible]
P(I)=0.3[illegible](EXP([illegible]))*COS(A[illegible])
P(I)=(P(I)+[illegible]*EXP(RSQ[illegible]))*(A[illegible]-AI)/A[illegible]
DO 19 [illegible]
[illegible]*(10.0**(LI-1))
[illegible]=3.1132*FLOAT([illegible])
[illegible]=6.28*[illegible]/540.[illegible]
WDT=6.28*FD/540.[illegible]
DO 7 N=1,NOP
Y(N)=P(N)*S[illegible]
[illegible]
[illegible]=0.0
[illegible]=3.14*F0*X
DO 17 J=1,27
AJ=[illegible]
FI=[illegible]
IF (J.GT.1) GO TO 77
SINC1=1.0
GO TO 7[illegible]
```

```
PROGRA  OP44.FOR
THIS IS AN EFFICIENT PROGRAM FOR OPTIMUM
PA. PERFORMANCE EVALUATION.HERE FINIT
SHOULD BE CHOSEN SO THAT IT IS INTEGRAL
SUB MULTIPLE OF TARGET DOPPLER SHIFT
AS WELL AS PRF/(NO. OF PULSES-1).THIS
CANNOT BE ACHIEVED FOR ALL DOPPLERS.
THIS TAKES ABOUT 30 SECS. FOR
COMPUTING DELTA ONCE
FINIT IS THE SAMPLING INTERVAL USED.
COMPLEX RES,RE,ST,UST
REAL X,Y,Z,P,Q
DIMENSION  X(20),Y(20),Z(20),P(20),Q(20)
DIMENSION RE(10),ST(20),UST(20),V(140),U(140)
T=10.0**-6
PI=22.0/7.0
NOP=4
AM=NOP
TNO=4.0*(10.0**6)/540.0
TTP=(10.0**6)/540.0
FINIT=20.0
NO=TNO
FNO=1.0
SN1=0.01
SIG=10.0
FD=280.0
H=FINIT
Maximum values of A,B,D, are given y AA,BB,BRD
AA=
BB=      (PI*540.0*0.033)**2
BRD=2.  (PI*540.0*0.044)**2
DO 350
AL
DO 350
DO 350
FD=FD+2
AJ
A=AA*AJ
BD=
PRINT
PRINT
PRINT
PRINT
FOR
FOR
FOR

TP=1.
HTP1=
DTP1=
ATP=
DO 11 I=1,NOP
AI=I-1
ASQI=AI*
RSQI=1.0*(       *DTP1)
DSQI=1.0*AI*
P(I)=0.  *(EXP(DSQI))*COS(AI*ATP)
P(I)=(P(I))+0.  *EXP(DSQI))*(AM-AI)/A
DO 10
SN=SN1*(10.0**(II-1))
PIFT=PI*FINIT*T
HTP=PI*2.0*FINIT/540.0
HDTP=PI*2.0*FD/540.0
DO 7
Y(N)=P(N)*SN
TO=0.0
FNDC=0.0
PIFDT=PI*FD*T
```

```
      DO 1030 J=1,27
      AJ=J-1
      FT=PIFT*AJ+PIFDT
      TPWD1=TTP*AJ
      UST(1)=(1.0,0.0)
      ST(1)=UST(1)
      DO 1010 I=2,NOP
      AI=I-1
      C00=TPWD1*AI
      C0=COS(C00)
      C1=SIN(C00)
      ST(I)=CMPLX(C0,C1)
1010  UST(I)=CONJG(ST(I))
      DO 103 K=2,NOP
      UST(1)=UST(1)+UST(K)
103   ST(1)=ST(1)+ST(K)
      U(1)=UST(1)*ST(1)
      SUM=0.
      FTP=PI*TTP
C     summation [illegible]
C     exp(-([illegible]).I.I.[illegible].TP)[illegible]COS(().(F+F1)+COS(().(F-F1))
C     found for I=[illegible]
      FTP1=2.[illegible]
      DO 11 I=1,NOP
      AI=I-1
      ARG=AI*FTP1
      S1=COS(ARG)
      S=X(I)*S1
11    SUM=SUM+S
1030  V(J)=SUM
      I=1
      DO 13 J=1,27
      AI=I-1
      AJ=J-1
      AF=AI*27.0+AJ
      FT=PIFT*AF+PIFDT
      IF (AF.GT.0.00) GO TO 77
      SINC1=1.0
      GO TO 78
77    FT1=FT-PIFDT
      SINC1=SIN(FT1)/FT1
78    SINC1=SINC1*SINC1
110   SINC=(SIN(FT))/FT
111   SINC=SINC*SINC
      SUM=V(J)*SINC
      SI=SINC1*U(J)
      ENOC=ENOC+SI
13    TO=(SI/(SUM+ENO))+TO
      DO 12 I=2,NO
      DO 12 J=1,27
      AI=I-1
      AJ=J-1
      AF=AI*27.0+AJ
      FT=PIFT*AF+PIFDT
      FT1=FT-PIFDT
      SINC1=SIN(FT1)/FT1
      SINC1=SINC1*SINC1
      SINC=(SIN(FT))/FT
      SINC=SINC*SINC
      SUM=V(J)*SINC
      SI=SINC1*U(J)
      ENOC=ENOC+SI
12    TO=(SI/(SUM+ENO))+TO
      TYPE *,II
      ENOC=ENOC*SIG*2.0*T*H/ENO
      TO=TO*SIG*2.0*T*H
C     SI contains the value of sinc sq.(PI.F.T)
C     for various values of F
C     TO contains Snc(F)
19    PRINT 104,JJ,SN,TO,ENOC,FD
350   CONTINUE
      STOP
      END
```

```
      PROGRAM DIS.FOR
      This progm. prepares data for autocorrelation plot
      DIMENSION P(32,8)
      NOP=16
      AA=4.0*3.14*0.033*540.0
      BB=8.0*(3.14*540.0*0.033)**2
      CC=2.0*3.14*0.1*540.0
      BBD=2.0*(3.14*540.0*0.044)**2
      ACCEPT *,N1,N2,N3,N4,N5,N6,N7,N8,N9
      N1,N2 GIVES THE RANGE OF VALUES OVER
      WHICH THE PARAMETER A WILL BE VARIED
      N3,N4-RANGE FOR D
      N5,N6-RANGE FOR A
      N7,N8-RANGE FOR C
      M=0
      DO 10 L=N1,N2
      AL =L-1
      B=BB*AL/8.0
      DO 10 J=N3,N4
      AJJ=J-1
      BD=(BBD*AJJ/4.0)+B
      DO 10 K=N5,N6
      AK=K-1
      DO 10 N=N7,N8
      AN=N-1
      C=CC*AN/8.0
      A=AA*AK/8.0
      TP=1.0/540.00
      BTP1=B*TP*TP
      DTP1=BD*TP*TP
      ATP=A*TP
      CTP=C*TP
      M=M+1
      DO 10 I=1,NOP
      AI=I-1
      BSQT=AI*AI
      BSQT=-1.0*(   BSQT*BTP1)
      DSQ1=-1.0*AI*DTP1*AI
      P(I,M)=0.33*(EXP(DSQT))*COS(AI*ATP)
      P(I,M)=P(I,M)+0.66*EXP(BSQT)
10    P(I,M)=P(I,M)*COS(CTP*AI)
101   FORMAT(1X,I3,8F10.5)
      LARGEST,SMALLEST VALUE OF YAXIS COMPUTED
      SMALL=0.0
      BIG=0.0
      DO 11 I=1,16
      DO 11 J=1,8
      IF(BIG.LT.P(I,J)) BIG=P(I,J)
      IF(SMALL.GT.P(I,J)) SMALL=P(I,J)
11    CONTINUE
      X1=SMALL
      SIGMAR=0.044*540.0*AJJ/8.0
      SIGMAD=0.033*540.0*AL/8.0
      DRFREQ=0.033*540.0*AK/8.0
      DPFREQ=C/6.28
      X2=BIG
      Y1=0
      Y2=15
      PRINT *,X1,X2
      PRINT *,Y1,Y2

      DO 100 I=1,16
      PRINT 101,I,(P(I,K),K=1,8)
100    CONTINUE

      GO TO (103,102,104,50) N9
102   PRINT 1020,SIGMAD,DRFREQ,DPFREQ
1020  FORMAT(1X,' VARYING DROTATION VAR.',
     2X, 'SIGMAD=',F5.2,1X,'DRFREQ=',F5.2,1X,'DPFREQ=',F5.2)
      GO TO 7
103   PRINT 1030,SIGMAR,DRFREQ,DPFREQ
1030  FORMAT(1X,' VARYING SIGMAD',2X,
     1'SIGMAR=',F5.2,1X,'DRFREQ=',F5.2,2X,'DPFREQ=',F5.2)
```

```
      GO TO 7
104   PRINT 105,SIGMAD,SIGMAR,DPFREQ
105   FORMAT(1X,' VARYING DRFREQ.MEAN',
    3 2X,'SIGMAD=',F5.2,2X,'SIGMAR=',F5.2,1X,'DPFREQ=',F5.2)
      GO TO 7
50    PRINT 51,SIGMAD,SIGMAR,DRFREQ
51    FORMAT(1X,' VARYING DUP.FREQ.MEAN',2X,
    1 'SIGMAD=',F5.2,2X,'SIGMAR=',F5.2,2X,'DRFREQ=',F5.2)
      PRINT 106
106   FORMAT(5X,'NORMALIZED DELAY',20X,'CORRELATION')
      CALL GRAPH
      STOP
      END
```

```
FIRST DIMENSION OF Y=DIMENSION OF X
SECOND DIM. OF Y=No.OF GRAPHS PER PAGE
IN THE INPUT FILE NAMED FORLPT.DAT THE
1st LINE SHOULD HAVE THE Min. VALUE OF
X,MAX.VALUE.2nd LINE SHOULD HAVE YMin,
YMAX..
THE LAST BUT ONE LINE SHOULD HAVE THE
TITLE OF THE GRAPH IF ANY.THE LAST
SHOULD HAVE TITLE FOR XAXIS,10 BLANKS,
TITLE FOR YAXIS
DIMENSION X(11),Y(11,4),A(158),IMAG4(5151)
OPEN(UNIT=6,DEVICE='DSK',FILE='RAMES.DAT')
OPEN(UNIT=22,DEVICE='DSK',FILE='FORLPT.DAT')
READ(22,*)A(147),A(148)
READ(22,*)A(145),A(146)
DO 44 I=1,11
READ(22,*) X(I),(Y(I,J), J=1,4)
4      CONTINUE
READ(22,10)(A(I),I=1,144)
0      FORMAT(72A1)
A(149)=0.0
CALL USPLX(X,Y,11,4,1,11,A,IMAG4,IER)
CONTINUE
CLOSE(UNIT=6,DEVICE='DSK',FILE='RAMES.DAT')
CALL UERTST
CALL USMNMX
9      STOP
END
```